Summary Method C3 (Jun '12)

1.
Apply the mid-ordinate rule from the formula book, defining x_0 to x_3 and then using $x_{0.5}$, $x_{1.5}$ and $x_{2.5}$ to find the corresponding y ordinates.

2.
a)
Begin to solve simultaneously by eliminating y, and rearrange to produce an equation of the form $f(x) = 0$. Then demonstrate that $f(0.5)$ and $f(1.5)$ lie either side of zero (one positive, one negative).
b)
Divide by 4, then use the fact that e^x is the inverse function of $\ln x$ to complete the rearrangement.
c)
Substitute in the value given for x_1 to calculate x_2, and use your value for x_2 (in as accurate a form as possible) to find x_3. Then give answers to required level of accuracy.
d)
Draw a line from x_1 up to the curve, then horizontally from that point to the line $y = x$. A dotted line vertically downwards from here gives the value of x_2 on the x-axis. From this point, draw a line vertically to the curve once again, and again horizontally to meet the line $y = x$. Repeat until all x values are represented.

3.
a)
Use the product rule (first term × differential of second + second term × differential of first).
b)
i.
Substitute $x = e$ into your expression for $\frac{dy}{dx}$ to find the gradient of the tangent at that point. Substitute $x = e$ into the original function to find the co-ordinates of the point, and use $y - y_1 = m(x - x_1)$ to find the equation of the tangent at that point.
ii.
Substitute $y = 0$ into your equation for the tangent line, and solve. Note: an exact value is required, so leave in terms of e^k where appropriate.

4.
a)
Set $u = x$ in order to simplify the result. Use the Integration by Parts formula from the formula book.
b)
Recall the required integral for a volume of revolution about the x-axis: $V = \int \pi y^2 \, dx$. After simplifying, this should closely resemble the integral from part a), and be readily calculable. Ensure your answer is left in the form specified.

5.

a)

Recall that $\sqrt{x}$ cannot be negative (otherwise it would be written as $\pm\sqrt{x}$).

b)

i.

Recall that $fg(x) = f(g(x))$. Substitute the function $g(x)$ into the function $f(x)$ in place of x.

ii.

Set your answer to b)i. equal to 5, rearrange and solve for x.

c)

i.

Set $f(x) = y$, then rearrange to find x in terms of y, then interchange x and y. Replace y with $f^{-1}(x)$.

ii.

Set your answer to c)i. equal to 7, rearrange and solve for x.

6.

Find $\frac{du}{dx}$ and use this to replace dx, along with part of the numerator. Replace any instances of $x^4 + 2$ with u, and replace any other instances of x with an equivalent expression in u. Make sure you give the answer in the required form.

7.

a)

Recall that $|f(x)| = f(x)$ when $f(x) \geq 0$ and $|f(x)| = -f(x)$ when $f(x) < 0$.

b)

Consider negative values of x, and how they will be interpreted by the function $f(|x|)$.

c)

Recall that $af(x)$ represents a stretch in the y-direction with scale factor a, and $f(x+b)$ represents a translation of $\begin{bmatrix} -b \\ 0 \end{bmatrix}$.

d)

Use the sequence of transformations you have described to find the new maximum point. Note that a stretch in the y-direction will multiply the distance of each point from the x-axis by the scale factor.

8.

a)

Adding the fractions using the standard common-denominator method will yield expressions in both the numerator and denominator which simplify using $\sin^2\theta + \cos^2\theta = 1$.

b)

Use the proven simplification from part b) to rewrite this equation in terms of $\operatorname{cosec}^2\theta$, then rearrange and solve for $\sin x$, and finally for x. Make sure answers are given from the required range and to the required level of accuracy,

9.

a)

The quotient rule is given in your formula book.

b)

A variation on $\sin^2\theta + \cos^2\theta = 1$ links $\tan\theta$ and $\sec\theta$.

c)

Rearrange to give x in terms of y, find $\frac{dx}{dy}$ using the result from part a), then use the result that $\frac{1}{\frac{dx}{dy}} = \frac{dy}{dx}$, along with the result from part b), to prove the required result.

d)

i.

Differentiate to find $\frac{dy}{dx}$ using the result from part c). Set the resulting expression equal to 0, rearrange and solve to find x ordinates for stationary points.

ii.

Use quotient rule to differentiate your expression for $\frac{dy}{dx}$ from part d)i.

iii.

Substitute your x values for stationary points into your expression for $\frac{d^2y}{dx^2}$ from part d)ii. to find any which are minimum $\left(\frac{d^2y}{dx^2} > 0\right)$. Confirm that this value of x corresponds to a value of $y = 0$ to show that the point lies on the x-axis.