Summary Method C3 (Jan '08)

1.
a)
i.
Use the chain rule to differentiate the expression. Recall $\left(f(x)\right)^n$ differentiates to $n\left(f(x)\right)^{n-1}f'(x)$.
ii.
Use the product rule for differentiation. The product of each term with the derivative of the other, added together.
b)
Use the quotient rule. (Bottom × Differential of Top – Top × Differential of Bottom) ÷ Bottom^2.

2.
a)
Rearrange to turn $\cot x$ into $\tan x$, then, after finding the primary solution on your calculator, add 180° (or π^c) for subsequent solutions within the range.
b)
Use the identity connecting $\cot x$ and $\operatorname{cosec} x$ obtainable from $\sin^2 x + \cos^2 x = 1$.
c)
Solve the second version of this identity as a quadratic, if necessary replacing $\cot x$ with y.

3.
a)
Substitute the values given for x and demonstrate that one gives a positive result, one a negative.
b)
Subtract x from both sides, then rearrange to make the x on the left-hand side the subject.
c)
Substitute the value given to find x_2, then x_3 then x_4.

4.
a)
Recall the range of a function is the possible values it can take (this may be limited by the form of the function – eg e^x cannot be negative – or due to the choice of domain.
b)
i.
The function $fg(x)$ is simply the function $g(x)$ substituted for x in $f(x)$.
ii.
Rearrange and solve.
c)
i.
To find an inverse, make the function equal to a new variable, say y, then make x (or the old variable) the subject. At the end, switch x and y.

5.
a)
i.
Differentiate the expression. Recall ax^n differentiates to anx^{n-1}.
ii.
Recall that $\int \frac{f'(x)}{f(x)} = \ln f(x)$ (this is in the formula book). Use log rules if necessary to simplify.
b)
Replace the $3x - 1$ by u in the integral, and also find $\frac{du}{dx}$ in order to remove dx and insert du. This should make the integral more straightforward to calculate.

6.
a)
Recall that $\operatorname{cosec} x = \frac{1}{\sin x}$, so use your knowledge of the $y = \sin x$ graph to convert (eg, where $y = 0$ on the $\sin x$ graph there will be an asymptote on the new one, where $y = 1$ on the $\sin x$ graph, $y = 1$ on the new one. For $y < 1$ on $y = \sin x$, $y > 1$ on the new graph, etc.
b)
The formula for the mid-ordinate rule is in the formula book. Remember to write out each x ordinate, then the mid-ordinates (half a step between), then the y mid-ordinates. Note: Always write down the full value on your calculator display as well as saving them in the memory to minimise error.

7.
a)
Recall that vertical stretches and translations involve multiplying the whole function by something or adding something to the whole function. Horizontal stretches and translations involve dividing each instance of x by something, or subtracting something from each instance of x. The order in which these things are done is frequently important, so double check.
b)
Start by sketching the quadratic $y = 4x^2 - 5$ using, if it helps you, your description of the graph transformations from a. Then reflect anything below the x axis above. Make sure you mark on all crossing points.
c)
i.
Use your graph to identify which non-modulus lines produce which crossing points, equate them and solve. Alternatively, write out the two possible non-modulus equations making a note of their limits of validity, and solve.
ii.
Use the graph to identify the region(s) within which the modulus graph is higher than the line $y = 4$, and use your critical values from part i to produce the solution range.

8.

a)

Rearrange by reversing the processes. Recall that the inverse operation to e^x is $\ln x$, so $e^{\ln x} = \ln e^x = x$.

b)

Set $u = x$ in the parts formula (in the formula book), so that when it is differentiated the new integral will be simpler.

c)

i.

Stationary points are defined as points where the gradient, $\frac{dy}{dx}$, is equal to 0. Find the differential (recall that e^x differentiates to e^x and use the chain rule). Leave your answers in terms of e.

ii.

The nature of a stationary point can be determined by finding $\frac{d^2y}{dx^2}$, substituting known values of x into it and determining whether it is positive or negative. A positive answer suggests a minimum (gradient is increasing), and a negative suggests a maximum (gradient is decreasing). An answer of 0 would imply a point of inflection (neither maximum or minimum).

iii.

Recall the formula for a volume of revolution (*not* given in the formula book): $\int \pi\, y^2 dx$. Calculate y^2 in terms of x, substitute in and integrate. Note: you should be able to use previous results to simplify your solution.