Summary Method C1 (Jun '13)

1.

a)

If a particular point lies on a line (or a curve, for that matter), the x-coordinate can be substituted into the equation instead of x, and the y-coordinate instead of y. Since the point is on the line or curve, the equation of the line or curve will still be valid. Any unknown values (in this case, p) can be found by solving the resulting equation.

b)

Gradient can be found easily from any line equation by simply rearranging into the form $y = mx + c$. The constant in front of the x (the x coefficient) will be the gradient. Take care not to make mistakes with the minus sign.

c)

The product of the gradients of any two perpendicular lines is always -1, so use the gradient found in part b) to write down the gradient of AC. Then use the point $(1,2)$ and the gradient you've just written down to find the equation of the line AC. Finally, substitute in the x and y coordinates of the point C and solve the resulting equation to find k.

d)

To find where two lines intersect, solve their line equations simultaneously. Usually the elimination method is the best choice. The resulting x and y values are the coordinates of the point of intersection.

2.

a)

i.

Find a square factor of the number inside the square root and separate using $\sqrt{a^2b} = a\sqrt{b}$.

ii.

Simplify each surd as far as possible – this should mean they all become multiples of $\sqrt{3}$. Then rearrange and solve the equation, remembering that you can divide by $\sqrt{3}$ in the same way you can divide by any other non-zero number.

b)

Rationalise the denominator by multiplying the top and bottom of the fraction by $2\sqrt{3} - \sqrt{5}$. This will eliminate any surds from the bottom and allow you to divide each part of the top (when simplified) by the rational number on the bottom. Remember to give your answer in the form required, using the result $\sqrt{a}\sqrt{b} = \sqrt{ab}$ where necessary.

3.

a)

Complete the square separately with x parts and y parts, then simplify further by moving all numbers to the right hand side of the equation.

b)

i.

The centre will be the x and y values needed to make the brackets in your equation equal to zero. This means the x and y coordinates of the centre will be the opposite sign to the numbers in the brackets.

ii.

The radius is always the square root of the number on the right hand side when in this form.

c)

i.

When sketching a circle, start with the centre, then use the radius to determine the top, bottom, left and right extremes of the circle. This will enable you to determine where the circle will lie and – importantly – which axes it crosses or touches. If you can't sketch a shape that looks roughly like a circle, bring a compass and use that.

ii.

It is possible to do this algebraically, but it is much easier to use your sketch to investigate where on the circle you would be furthest from the x axis.

d)
It would be a good idea to sketch this circle onto your original axes, or at least mark on the centre point. Then – since the radius is unchanged – note that the transformation will simply be a translation and describe it either in words (how far right/left, how far up/down) or using a translation vector (recalling that the first element will indicate how far to the right and the second how far up – if you want left or down, use a negative). It is important to note which circle you are translating to which.

4.
a)
i.
It is a good idea to quote the factor theorem, but remember that it works both ways:

> *"If -3 is a root, then $(x + 3)$ is a factor" is true, but so is "If $(x + 3)$ is a factor, -3 is a root". These statements only state the result in one direction, which doesn't automatically make the other one true. For instance "If James is not a father, then he has no daughters" is, of course, true. "If James has no daughters, then he is not a father" is clearly not. In the case of the factor theorem, to ensure you have fully stated it, either use the double-implied arrows $\Leftrightarrow$ (for instance "James is not a father $\Leftrightarrow$ James has no children"), or the maths word IFF which means 'if and only if' (eg "Iff James is not a father, then James has no children). Either of these indicates that the statement works both ways round.*

Once you have demonstrated that $f(-3) = 0$ (which is completely equivalent to saying that -3 is a root), you need to remember to make the statement "Therefore $(x + 3)$ is a factor".
ii.
Use inspection to determine firstly the value of q in the quadratic factor, then – by considering the x^2 or x coefficients in the final cubic (or, to be on the safe side or to check an answer, both), determine the required value of p.
b)
i.
Differentiate using the rule $x^n \longrightarrow nx^{n-1}$, and recalling that constants differentiate to zero. And – since we're not integrating – there's no need for a constant of integration!
ii.
You must make the statement "At stationary points, $\frac{dy}{dx} = 0$" to justify forming the equation, then simplify (one simple step) to generate the equation required. Even though this is a simple question, take care that you have clearly laid out your method and made your reasoning clear. Just writing out the equation you have been asked to generate proves nothing.
iii.
The key aspect of this question is the claim that 'the only stationary point occurs when $x = -3$". Since we have already factorised the cubic in question and know that -3 is a factor, it is trivial to note that $x = -3$ gives a stationary point. What this question requires is that you demonstrate that there are no others. In other words, the solution to the cubic equation found from the linear factor $(x + 3)$ is the only solution. That is, the quadratic factor can never equal zero, and therefore the discriminant of this quadratic will be negative. Demonstrate this, make a statement explaining why this rules out other stationary points and you're done.
iv.
Note that you need to differentiate your expression for $\frac{dy}{dx}$, not the version from the equation after you divided everything by 4. This is not the same as $\frac{dy}{dx}$, it was simply a step along the way after equating your expression to 0. Differentiate, substitute in the value $x = -3$ and evaluate.
v.
Recall that at any points on the curve where $\frac{d^2y}{dx^2} > 0$, the gradient is increasing. If this also occurs at a stationary point (where $\frac{dy}{dx} = 0$), then this is an indication that the stationary point must be a minimum (the gradient is increasing, so before it was zero it had to be negative – the graph going down – and after it is zero it will be positive – the graph going up. The only shape that fits is a local minimum).

5.

a)

i.

To complete the square for a quadratic with an x^2 coefficient other than 1, you need to take out the x^2 coefficient as a factor of the whole quadratic, turning the x coefficient and constant term into fractions if necessary. Then fully complete the square with the quadratic inside the bracket, and finally put the factor back in, writing it in front of the squared bracket and multiplying it by the additional term at the end. Alternatively, multiply out the expression with p and q in it and compare it directly to the quadratic in question to determine the necessary values of p and q.

ii.

Recall that any minimum or maximum point for an expression in completed square form can be found by setting the squared bracket equal to zero (the lowest result possible for a squared number). When this bracket has a negative number in front, the result will be a maximum, but in this case a minimum.

b)

i.

AB^2 means the distance between A and B squared (presumably because the method involves Pythagoras' theorem, and so unless it was squared there would be a square root involved in the final answer). Use Pythagoras' theorem with the difference between the x coordinates and the difference between the y coordinates as the two shorter sides of the triangle. Multiply out the brackets and simplify to get the required expression.

ii.

You already have the minimum value for $2x^2 + 6x + 5$, so you can find a minimum for AB^2 and therefore for AB. Remember that $\sqrt{\frac{a}{b^2}} = \frac{\sqrt{a}}{b}$.

6.

a)

To find the gradient of the curve at the point P, differentiate the equation of the curve and substitute in the x coordinate of P. Then use $y - y_1 = m(x - x_1)$ to find the equation of the tangent. Finally, rearrange to the desired form.

b)

i.

Since Q lies on the curve, substituting in the x and y coordinates to the curve equation should give a valid equation. Solve to find k.

ii.

To verify that Q lies on the tangent it is necessary to substitute *one* of the x or y coordinates of the point into the tangent line equation and solve to find the corresponding y or x value. If this is the same as the y or x coordinate of the point, you have demonstrated it lies on the line. It is not enough to simply substitute both coordinates in and fail to find a contradiction. By substituting in both you are making the assumption that the point is on the line, and therefore you cannot prove that it is.

c)

i.

Integrate the expression using the fact that ax^n integrates to $\frac{a^{n+1}}{n+1}$ and a constant a integrates to ax. You do not need a constant of integration because this is definite integration (with limits). Apply the limits, taking care with negatives.

ii.

The shaded region can be found by subtracting the integral found in c)i. from the area of the trapezium formed by the tangent line, the x axis and the lines $x = -1$ and $x = 2$. The area of the trapezium can be found either by using the formula $A = \frac{a+b}{2}h$ or integrating the equation of the tangent between the same limits as the curve.

7.

a)

Recall that for a quadratic to have no solutions the discriminant $b^2 - 4ac$ must be negative, for it to have one solution the discriminant must be zero and for two solutions it must be positive. This means that for a quadratic to have 'real roots' – that is, either one or two solutions, the condition is $b^2 - 4ac \geq 0$. The question itself is a good indicator that we will be dealing with either $\geq$ or $\leq$ rather than simply $>$ or $<$ given the inequality we are asked to prove. Substitute the x^2 and x coefficients and the constant into the condition inequality $b^2 - 4ac \geq 0$, rearrange and simplify (remembering that if you choose to divide or multiply both sides by a negative number you will have to swap round the inequality sign to compensate).

b)

Note that the quadratic with real solutions is the original quadratic in x. The quadratic inequality we are now examining, in k, will tell us, if we can solve it, the values of k for which the original quadratic has real roots. Write out the inequality as an equation $(= 0)$ and solve to find critical values for k. In this case, the quadratic will factorise but you can also use the formula or completing the square. Once you have critical values, a quick sketch of the graph will enable you to determine whether we need to be between the two critical values in order for the whole quadratic expression to be ≤ 0 or either side of the critical values. Finally write out your solution (an inequality) for possible values of k.