Summary Method C1 (Jan '13)

1.
a)
i.
Substitute the x coordinate and y coordinate of the point B into the equation of the line AB and solve the resulting equation to find k.
ii.
Midpoint is the average of the x coordinates and the average of the y coordinates.
b)
Gradient is $\frac{y-step}{x-step}$ or $\frac{y_2-y_1}{x_2-x_1}$. Careful with negatives.
c)
The gradients of two perpendicular lines will satisfy $m_1m_2 = -1$. That is, the gradient of the perpendicular to AB can be found by flipping the fraction from part b) and changing the sign. Use the general form of the line equation $y - y_1 = m(x - x_1)$ to find the equation of the line with gradient m passing through point (x_1, y_1).
d)
The intersection of two lines is the solution of the simultaneous equations. Rearrange and use elimination (or, if you prefer, substitution) to solve simultaneously and find x and y.

2.
a)
Differentiate the function. Remember that ax^n differentiates to anx^{n-1}.
b)
i.
Rate of change is the same as the differential with respect to time (which you have just calculated). Substitute in the value $t = 1$.
ii.
If the rate of change is positive, it means the height is increasing (consider a graph with a positive gradient), and vice versa.
c)
i.
Differentiate your answer to a), and substitute in the value $t = 2$.
ii.
Recall that if the second derivative is positive, this means the gradient of the original function is itself increasing. For a stationary point, this means it has gone from negative to zero to positive, and therefore represents a minimum. The converse is true for a negative second derivative.

3.
a)
i.
Simplify by finding a square factor of 18, and using the fact that $\sqrt{a^2 \times b} = a\sqrt{b}$.
ii.
Simplify each part to write as surds in their simplest form, combine where possible then use rules of fractions to simplify your overall answer. You can multiply or divide both numerator and denominator by any number, but you cannot add.
b)

Multiply top and bottom by the conjugate to give the difference of two squares. That is, by $2\sqrt{2}+\sqrt{3}$. Then simplify. Recall that $2\sqrt{5}\times 3\sqrt{5} = 2\times 3\times 5$, not $2\times 3\times 25$.

4.
a)
i.
Complete the square, remembering to combine the subtracted squared component and the original $+11$.
ii.
You must use the result above – no marks will be given for alternative methods (such as using the discriminant or attempting to apply the quadratic formula). A quadratic in completed square form can be rearranged to give $x = \cdots$. If this requires you to square root a negative, there will be no solutions.
b)
i.
The vertex is the minimum (or, in the case of a negative quadratic, maximum) point. This can be found from the completed square form by considering the value of x which makes the squared bracket equal to zero, and the resulting value for y.
ii.
Find the y intercept by setting x equal to zero (in the original form will be simplest, but either will of course work). Use the vertex found in b)i. to complete your sketch.
iii.
Refer to the completed square form to determine how the curve has been transformed. In this case, it is a translation in both the x and y directions. Note the direction (what maps our curve onto $y = x^2$, not the other way around), and use the vertex to determine how far the curve must move in each direction.

5.
a)
The remainder theorem states that when $P(x)$ is divided by $(x-a)$ the remainder will be $P(a)$.
b)
i.
The factor theorem states that if $(x-a)$ is a factor of $P(x)$ then $P(a) = 0$ (that is, a is a root).
ii.
Use the linear factor from b)i. and use the inspection method to find the quadratic factor that goes with it. Since the question requires a product of linear factors, this quadratic should factorise into two linear factors.
c)
The roots of the equation (which can be determined from the three factors already found) are also the x coordinates of the points where the curve intersects the x axis. To determine the y intercept, substitute $x = 0$ into the original function. Note that this is a positive cubic, so it will start below the x axis and end above, with up to two stationary points (as in this case) in between. It is not necessary to determine the position of local minima or maxima for this sketch (although this would be achievable by differentiation if required).

6.

a)

Note that the gradient of the tangent to the curve at a given point must be equal to the gradient of the curve at that point (this is the definition of the gradient of a curve). Since you are given an expression for $\frac{dy}{dx}$ there is no need to differentiate. Simply substitute in the x coordinate of the point in question to find the gradient. To determine the equation of the line, use $y - y_1 = m(x - x_1)$.

b)

To determine the equation of the curve you need to work backwards from the derivative $\frac{dy}{dx}$. This means integration, but to find the complete equation you also need to use the fact that the point (1,4) lies on the curve. Integrate, include the constant of integration C, then determine its value by substituting in the x and y values of that point. Finally, rewrite with the now known value of C.

7.

a)

y intercepts for any curve can be found by substituting in $x = 0$ (since the line $x = 0$ lies exactly on the y axis). In this case, it will require you to solve the resulting quadratic equation in y. Always try factorising first, but if that doesn't work, you can use the quadratic formula or complete the square to find values.

b)

To read off the radius of a circle from its equation, write it in the form $(x - a)^2 + (y - b)^2 = R^2$. For a circle in this general form, the radius is R, and the centre is at (a, b), so you can check your answer to some extent by verifying that the centre is indeed $(-3,2)$ as given in the question. To rearrange, complete the square separately for x and x^2 parts, then for y and y^2 parts. Finally, don't forget to square root the number on the right hand side of the equation, since it represents R^2, not simply R.

c)

i.

The distance between two points can be calculated by sketching a right angled triangle whose hypotenuse is the line between them, and using Pythagoras' theorem with the difference between the x coordinates and the difference between the y coordinates as the lengths of the two shorter sides.

ii.

Draw a sketch, and use the fact that a tangent meets the radius of the circle at the point of contact at 90°. This means you can calculate PQ using the distance CP (found in c)i.) and the length of CQ (which, it should be clear, is the same as the radius of the circle). Apply Pythagoras' theorem.

8.

a)

To find any points of intersection it is necessary to solve the equations simultaneously. In this case, this is most readily accomplished by substituting for y.

b)

i.

Use the criteria that the discriminant $b^2 - 4ac > 0$ for two distinct roots. Note that the quadratic in question is in x, so values of a, b and c will involve numbers and k.

ii.

To solve the quadratic inequality it is necessary to first solve the equation formed by replacing the inequality with an $=$ sign. This will give critical values. By sketching the curve or examining the sign of the curve before, between and after the critical values it can be determined when the curve would be above the axis.