Summary Method C1 (Jan '08)

1.

a)

Midpoint is the average of the x coordinates and the average of the y coordinates.

b)

i.

Gradient is $\frac{y-step}{x-step}$ or $\frac{y_2-y_1}{x_2-x_1}$. Careful with negatives.

ii.

Equation of a straight line is given by $y - y_1 = m(x - x_1)$. Use either A or B, and the gradient calculated. Remember to rearrange to the required form.

iii.

Same straight line formula with the same gradient, but use coordinates of C.

2.

a)

Differentiate the function

b)

Set the differential equal to 0 and solve for x.

c)

i.

Differentiate your answer to a).

ii.

Substitute the x value found in b), and determine if the result is positive or negative. Positive means a minimum, negative a maximum.

d)

Substitute $x = 0$ into your expression for the gradient (part a). Positive means increasing, negative decreasing.

3.

a)

Multiply top and bottom of the fraction by $\sqrt{2}$, simplify $\sqrt{8}$ and then simplify the whole expression

b)

Multiply top and bottom by $3\sqrt{2} + 4$ to rationalise the denominator, then multiply out and simplify.

4.
a)
Complete the square separately on the x parts and the y parts, move constants to the right-hand side and simplify.
b)
i.
The general circle equation $(x-a)^2+(y-b)^2=r^2$ has centre (a,b) ...
ii.
... and radius r. Remember to leave in surd form – this is the only exact way of expressing the answer.
c)
i.
Substitute $2x$ for y in the circle equation and simplify.
ii.
Use the discriminant b^2-4ac to determine the number of solutions. No solutions implies the line doesn't cross the circle, two solutions that it crosses twice, one that it touches once, as a tangent.
d)
Find the distance between Q and the centre C using Pythagoras' Theorem, and show that it is less than the radius calculated in part b)ii.

5.
a)
Factorising can be made simpler in this case by taking out -1 as a common factor, factorising, then multiplying it back in.
b)
Multiply out the completed square form and simplify.
c)
i.
The line of symmetry will be a vertical line (ie, $x=k$) which passes through the vertex. The completed square form from part b) is sufficient to determine the vertex. $y=(x-a)^2+b$ has vertex (a,b) and thus line of symmetry $x=a$.
ii.
See above.
iii.
Use the completed square form (and your answers to i. and ii.) to determine the maximum (must be max rather than min since the coefficient of x^2 is negative). Use the standard form given in the question for part a) to determine the crossing point at the y axis, and the completed square form again to solve for x and give points of intersection with the x axis.

6.

a)

i.

The factor theorem states that if $x - a$ is a factor, a must be a root. Therefore $p(a) = 0$.

ii.

Use $(x + 1)$ as one of the factors, determine using inspection or algebraic long division the quadratic term which remains, and factorise this.

b)

i.

The crossing points on the x axis correspond to the roots, which in turn correspond to the factors found in part a) ii.

ii.

Integrate the expression and substitute the values into your final expression. Remember, no constant of integration is required (since it would cancel) for definite integration. Note that the curve is always below the axis here, so your answer should be negative.

iii.

This will be the absolute value (positive) of your answer for ii.

iv.

Differentiate the function and substitute the value $x = -1$.

v.

The normal has gradient $-\frac{1}{m}$ where m is the gradient of the tangent.

7.

a)

Substitute the line equation into the quadratic and simplify.

b)

Use the condition that the discriminant $b^2 - 4ac$ must be greater than 0 for two distinct solutions.

c)

Solve the quadratic as if equal to zero to find critical values, then, using a sign diagram or sketching a graph, determine the range(s) for which the expression is positive.