MM2B

Q	Solution	Marks	Total	Comments
1(a)	KE = $\frac{1}{2}\times 58\times 2^2$	M1		M1: Correct fully substituted expression for KE.
	= 116 J	A1	2	A1: CAO
(b)	Change in PE: $mgh = 58 \times 9.8 \times 7$	M1		M1: Expression for PE with 58 and 9.8 or 9.81 with 6 or 7 for the height (or 11 and 4, 11 and 5 or 10 and 4).
	= 3978.8	A1		A1: Accept 3980 or 3970 or 3978 or 3979 or 3978.8. Accept 3982 or 3983 or 3980.
	KE = 3978.8 + 116 J = 4094.8 J	M1		M1: Adding their two previous answers.
	Speed of Kim is $\sqrt{\dfrac{4094.8}{\frac{1}{2}\times 58}}$	dM1		dM1: Seeing expression for v (not v^2), dependent on second M1
	= 11.88 m s^{-1} = 11.9 m s^{-1}	A1	5	A1: Accept 11.88 or 11.8 or 11.9 Accept 11.88 or 11.8 or 11.9 or AWRT 11.89 from g = 9.81. Obtaining $v=\sqrt{u^2+2gh}$ followed by incorrect substitution M0M1M1, unless h is 6 or 7, which is M1M1M1 11.0 (from h = 6) M1M1M1 $v=\sqrt{2^2+2\times g\times 7}$ M1M1M1 $=\sqrt{141.2}$ A1 $=11.9$ A1 $v=\sqrt{4+14g}$ M1M1M1A1 $=11.9$ A1 $v=\sqrt{2^2+12g}$ M1M1M1
	Total		**7**	

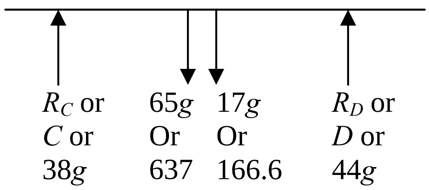
MM2B (cont)

Q	Solution	Marks	Total	Comments
2	$\bar{X} = \dfrac{2\times9+3\times2+8\times3+7\times6}{2+3+8+7}$	M1		M1: Expression for $\bar{X}$ with no more than one error in the numerator and correct denominator.
	$= \dfrac{90}{20}$ or 4.5	A1		A1: Correct distance. Accept $\dfrac{9}{2}$ or $\dfrac{90}{20}$ or equivalent.
	$\bar{Y} = \dfrac{2\times6+3\times4+8\times8+7\times11}{20}$	M1		M1: Expression for $\bar{Y}$ with no more than one error in the numerator and correct denominator.
	$= \dfrac{165}{20}$ or 8.25	A1		A1: Correct distance. Accept $\dfrac{33}{4}$ or $\dfrac{165}{20}$ or equivalent.
	$\therefore$ Centre of mass is at (4.5, 8.25)	A1F	5	A1: Correct coordinates; dependent on M1 M1 Do not accept $\dfrac{90}{20}$ etc at this stage. SC4: For final answer (8.25, 4.5) award 4 marks. Moments about *B*, (2.5, 4.25) SC2
	Total		**5**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\mathbf{a} = \frac{\mathrm{d}v}{\mathrm{d}t}$ $\mathbf{a} = -8\mathrm{e}^{-2t}\mathbf{i} + (6-6t)\mathbf{j}$	M1 A1 A1	3	M1: Differentiating with either of the two components correct. Do not need to see **i** or **j**. A1: Correct **i** component. A1: Correct **j** component.
(b)(i)	Using **F** = m**a** $\mathbf{F} = 5\times\{-8\mathrm{e}^{-2t}\mathbf{i} + (6-6t)\mathbf{j}\}$ $= -40\mathrm{e}^{-2t}\mathbf{i} + (30-30t)\mathbf{j}$	M1 A1	2	M1: Multiplying their acceleration by 5, even if not a vector. A1: Correct expression.
(ii)	Magnitude of **F** is $\{(-40)^2 + (30)^2\}^{\frac{1}{2}}$ = 50	M1 A1	2	M1: Finding magnitude from two non-zero terms. Must add terms and square root. Condone $\{(40)^2 + (30)^2\}^{\frac{1}{2}}$ A1: Correct answer only. In this part, condone lack of negative signs in expression for force in (b) (i).
(c)	When **F** acts due west, **j** component is zero $30 - 30t = 0$ $t = 1$	M1 A1	2	M1: Putting **j** component equal to zero. A1: Correct time.
(d)	$\mathbf{r} = -2\mathrm{e}^{-2t}\mathbf{i} + (3t^2 - t^3)\mathbf{j} + \mathbf{c}$ When $t = 0$, **r** = 6**i** + 5**j** ∴ **c** = 8**i** + 5**j** ∴ $\mathbf{r} = (8 - 2\mathrm{e}^{-2t})\mathbf{i} + (5 + \frac{1}{4}t^4 - t^3)\mathbf{j}$	M1 A1 A1 dM1 A1	5	M1: Integration with either of the two components correct. Do not need to see **i** or **j**. A1: Correct **i** component. A1: Correct **j** component. Condone lack of + **c** dM1: Finding **c** using 6**i** + 5**j** and $\mathrm{e}^0 = 1$. A1: Correct position vector.
	Total		**14**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
4(a)		B1 B1	2	B1: Two weights correct and in correct relative positions. B1: Two upward reaction forces, labelled differently. Note all forces must be shown as arrows and have labels. Condone use of $g = 9.81$ for calculating weights.
(b)	Taking moments about C $3 \times 17g + 2.6 \times 65g = 44g \times d$ $44d = 220$ $d = 5$ Distance is 5 – 4.6 = 0.4 m	B1 M1 A1 A1	 4	B1: Seeing 2.6. M1: Three term moment equation including $17g$, $65g$ and $44g$ or 17, 65 and 44, with different distances for the $17g$ and $65g$. A1: Correct equation. A1: Correct final answer.
	Alternative $R_C = 38g$ Taking moments about D $38g(4.6+x) = 65g(2+x) + 17g(1.6+x)$ $174.8 - 130 - 27.2 = 44x$ $x = 0.4$	 (B1) (M1) (A1) (A1)		Could take moments about any other point
(c)	Gravitational force (centre of mass or weight) at mid-point (or centre) of the plank	E1	1	E1: Correct explanation.
	Total		**7**	
5(a)	$90\,\text{km}\,\text{h}^{-1} = 90 \times \dfrac{1000}{3600}\,\text{m}\,\text{s}^{-1}$ $= 25\ \text{m s}^{-1}$ **AG**	B1	1	B1: Must see $\dfrac{1000}{3600}$ or $\dfrac{1000}{60^2}$.
(b)	Resistance is 5000 N Using power = force × velocity = 5000 × 25 = 125 kW	B1 M1 A1	 3	B1: Obtaining 5000. M1: Using $P = Fv$ with 25 and their F. A1: Correct final answer, must be in kW. 125W or 125 000 W B1M1 125 B1M1A1
	Total		**4**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
6(a)	Using $F = ma$ $-2mv^{\frac{5}{4}} = m\frac{dv}{dt}$ $\therefore\ \frac{dv}{dt} = -2v^{\frac{5}{4}}$ **AG**	B1	1	B1: Must see $-2mv^{\frac{5}{4}} = m\frac{dv}{dt}$ or $-2mv^{\frac{5}{4}} = ma$ and correct final answer.
(b)	$\int \frac{dv}{v^{\frac{5}{4}}} = -2\int dt$	M1		M1: Two integrals with one in the form $\int f(v)dv$ where $f(v) = v^{\pm\frac{5}{4}}$ or $v^{\pm\frac{4}{5}}$. The other integral must not contain v terms.
	$-\frac{4}{v^{\frac{1}{4}}} = -2t + c$	A1		A1: Correct expression. Condone lack of + c for this A1, but no subsequent marks if no c.
	When $t = 0$, $v = 16 \Rightarrow c = -2$	dM1 A1		dM1: Using $t = 0$ and $v = 16$ to find c. A1: Obtaining $c = -2$.
	$-\frac{4}{v^{\frac{1}{4}}} = -2t - 2$ $v^{\frac{1}{4}} = \frac{2}{1+t}$ $v = \left(\frac{2}{t+1}\right)^4$ **AG**	A1	5	A1: Correct final answer. Must see $v^{\frac{1}{4}} = \frac{2}{1+t}$ or $v^{-\frac{1}{4}} = \frac{1+t}{2}$ or $\frac{1}{v^{\frac{1}{4}}} = \frac{1+t}{2}$ Or if they obtain $v = \left(\frac{2}{t+c}\right)^4$ $v = 16,\ t = 0 \Rightarrow 16^{\frac{1}{4}} = \frac{2}{c}$, condone $c = 1$ (no other root considered)
	Total		**6**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
7(a)	Resolving vertically $T\cos 30 + 20\cos 50 = 4g$ $T\cos 30 = 26.344$ $T = 30.4$ N	M1A1 A1 A1	 4	M1: Three terms, which must include 4*g*, $T\cos\theta$ or $T\sin\theta$ and $20\cos\theta$ or $20\sin\theta$, where θ = 30, 40, 50 or 60. A1: Correct terms A1: Correct equation A1: Correct final answer. Accept 30.4 or AWRT 30.42. Accept 30.4 or 30.5 or AWRT 30.45 from $g = 9.81$.
(b)	Horizontally: $\frac{mv^2}{r} = 20\cos 40 + T\cos 60$ $\frac{4\times 5^2}{r} = 30.53$ $r = 3.27537$ $= 3.28$	M1 A1F dM1 A1	 4	M1: Three terms, which must include $\frac{mv^2}{r}$ or $\frac{4\times 5^2}{r}$, $T\cos\theta$ or $T\sin\theta$ and $20\cos\theta$ or $20\sin\theta$, where θ = 30, 40, 50 or 60. A1F: Correct equation. May include *T*, *m* and *v*. dM1: Substitution of values for *T*, *m* and *v*. Equation of form $\frac{4\times 5^2}{r}$ = number A1: Correct answer. Accept 3.27 or 3.28 or AWRT 3.28. Accept 3.27 or AWRT 3.27 from g = 9.81. **Note: Do not accept** $\frac{mv^2}{r} = 30.4$ **or similar.**
	Total		**8**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
8(a)	Using conservation of energy (lowest and highest points) $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(2a)$	M1A1		M1: Equation for conservation of energy with two KE terms and one or two PE terms. May see m or 0.3. A1: Correct equation.
	$u^2 = v^2 + 4ag$ For complete revolutions, $v > 0$ $\therefore u^2 > 4ag$ $u > 2\sqrt{ag}$ **AG**	A1	3	A1: Correct result with statement of $v > 0$ and some intermediate working including $4ag$ term.
	Or Use of PE at top and KE at B Correct PE and KE Correct deduction including inequality	 (M1) (A1) (A1)		
(b)(i)	C of Energy $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mga(1 + \sin\theta)$	M1A1		M1: Equation for conservation of energy with two KE terms and one or two PE terms including a $\sin\theta$. May see m or 0.3. A1: Correct equation.
	$v^2 = \left(\sqrt{\frac{9}{2}ag}\right)^2 - 2ga(1 + \sin\theta)$ $= \frac{5}{2}ag - 2ag\sin\theta$			
	Resolve radially $\pm R = -mg\sin\theta + \frac{mv^2}{a}$	M1A1		M1: Three term equation from resolving radially. Correct three terms, but condone signs and replacement of sin by cos. A1: Correct equation. May see m or 0.3.
	$= -mg\sin\theta + \frac{5}{2}mg - 2mg\sin\theta$ $= -3mg\sin\theta + \frac{5}{2}mg$			
	$= \left(\frac{3}{4} - \frac{9}{10}\sin\theta\right)g$ **OE** (must include g)	A1	5	A1: Simplified correct final answer. Condone $\left(\frac{9}{10}\sin\theta - \frac{3}{4}\right)g$
(ii)	When this reaction is zero, $\left(\frac{3}{4} - \frac{9}{10}\sin\theta\right)g = 0$	M1		M1: Putting their reaction equal to zero.
	$\sin\theta = \frac{5}{6}$ θ is 56.4° above horizontal	A1	2	A1: Correct angle. Accept AWRT 56.44.
	Total		**10**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
9(a)	EPE $= \dfrac{\lambda x^2}{2l}$			
	$= \dfrac{1800\times(4)^2}{2\times 6}$	B1 M1		B1: Extension = 4. M1: Substitution of 6, 1800 and their extension into EPE formula.
	$= 2400$ J	A1	3	A1: Correct EPE
(b)	$\dfrac{1800\times(x)^2}{2\times 6} = \dfrac{1}{2}\times 200\times 8^2$	M1		M1: Equation with EPE and KE terms, both correct.
	$x^2 = 42.67$ $x = 6.53$ m	A1		A1: Correct extension. Accept $\dfrac{8\sqrt{6}}{3}$ or 6.53 or AWRT 6.532.
	Distance from O is 12.5 m	A1	3	A1: Correct distance. Accept 12.5 or AWRT 12.53.
(c)	Resistance force is 800 N Work done by resistance force is $800 \times (x + 6)$	B1		B1: Correct work done by resistance force.
	C of Energy gives $\dfrac{1800\times(x)^2}{2\times 6} + 800\times(x+6) = \dfrac{1}{2}\times 200\times 8^2$	M1A1 A1		M1: Three energy terms, KE, Work Done and EPE. A1: EPE correct. A1: Correct equation.
	$150x^2 + 800(x+6) = 6400$ $3x^2 + 16x - 32 = 0$ or $150x^2 + 800x - 1600 = 0$	A1		A1: Correct quadratic equation with no brackets.
	$x = \dfrac{-16 \pm \sqrt{16^2 + 4\times 3\times 32}}{2\times 3}$	dM1		dM1: Solving their quadratic equation with correct formula and correct substitution
	$x = 1.5497$	A1		A1: Correct positive solution stated. Accept 1.54 or 1.55 or AWRT 1.55.
	Distance from O is 7.55 m	A1	8	A1: Correct distance from O. Accept 7.55 or 7.54 or AWRT 7.55.
	OR Use d for distance: $800 \times d$	(B1)		B1: Correct work done by resistance force.
	C of Energy gives $\dfrac{1800\times(d-6)^2}{2\times 6} + 800\times d = \dfrac{1}{2}\times 200\times 8^2$ $150d^2 - 1000d - 1000 = 0$ $3d^2 - 20d - 20 = 0$	(M1A1) (A1A1) (A1)		M1: Three energy terms, KE, Work Done and EPE. A1: Seeing $d - 6$ in EPE A1: EPE correct. A1: Correct equation. A1: Correct quadratic equation with no brackets.
	$x = \dfrac{-20 \pm \sqrt{20^2 + 4\times 3\times 20}}{2\times 3}$	(dM1)		dM1: Solving their quadratic equation.
	$d = 7.55$	(A1)		A1: Correct distance from O. Accept 7.55 or 7.54 or AWRT 7.55.
	Total		**14**	
	TOTAL		**75**	