General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS **MPC4**
Unit Pure Core 4

Thursday 11 June 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Use the Remainder Theorem to find the remainder when $3x^3 + 8x^2 - 3x - 5$ is divided by $3x - 1$. *(2 marks)*

(b) Express $\dfrac{3x^3 + 8x^2 - 3x - 5}{3x - 1}$ in the form $ax^2 + bx + \dfrac{c}{3x - 1}$, where a, b and c are integers. *(3 marks)*

2 A curve is defined by the parametric equations

$$x = \frac{1}{t}, \qquad y = t + \frac{1}{2t}$$

(a) Find $\dfrac{dy}{dx}$ in terms of t. *(4 marks)*

(b) Find an equation of the normal to the curve at the point where $t = 1$. *(4 marks)*

(c) Show that the cartesian equation of the curve can be written in the form

$$x^2 - 2xy + k = 0$$

where k is an integer. *(3 marks)*

3 (a) Find the binomial expansion of $(1 - x)^{-1}$ up to and including the term in x^2. *(2 marks)*

(b) (i) Express $\dfrac{3x - 1}{(1 - x)(2 - 3x)}$ in the form $\dfrac{A}{1 - x} + \dfrac{B}{2 - 3x}$, where A and B are integers. *(3 marks)*

(ii) Find the binomial expansion of $\dfrac{3x - 1}{(1 - x)(2 - 3x)}$ up to and including the term in x^2. *(6 marks)*

(c) Find the range of values of x for which the binomial expansion of $\dfrac{3x - 1}{(1 - x)(2 - 3x)}$ is valid. *(2 marks)*

4 A car depreciates in value according to the model

$$V = Ak^t$$

where £V is the value of the car t months from when it was new, and A and k are constants. Its value when new was £12 499 and 36 months later its value was £7000.

(a) (i) Write down the value of A. *(1 mark)*

(ii) Show that the value of k is 0.984 025, correct to six decimal places. *(2 marks)*

(b) The value of this car first dropped below £5000 during the nth month from new. Find the value of n. *(3 marks)*

5 A curve is defined by the equation $4x^2 + y^2 = 4 + 3xy$.

Find the gradient at the point $(1, 3)$ on this curve. *(5 marks)*

6 (a) (i) Show that the equation $3\cos 2x + 7\cos x + 5 = 0$ can be written in the form $a\cos^2 x + b\cos x + c = 0$, where a, b and c are integers. *(3 marks)*

(ii) Hence find the possible values of $\cos x$. *(2 marks)*

(b) (i) Express $7\sin\theta + 3\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. Give your value of α to the nearest $0.1°$. *(3 marks)*

(ii) Hence solve the equation $7\sin\theta + 3\cos\theta = 4$ for all solutions in the interval $0° \leqslant \theta \leqslant 360°$, giving θ to the nearest $0.1°$. *(3 marks)*

(c) (i) Given that β is an acute angle and that $\tan\beta = 2\sqrt{2}$, show that $\cos\beta = \frac{1}{3}$. *(2 marks)*

(ii) Hence show that $\sin 2\beta = p\sqrt{2}$, where p is a rational number. *(2 marks)*

Turn over for the next question

7 The points A and B have coordinates $(3,\ -2,\ 5)$ and $(4,\ 0,\ 1)$ respectively.

The line l_1 has equation $\mathbf{r} = \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$.

(a) Find the distance between the points A and B. *(2 marks)*

(b) Verify that B lies on l_1. *(2 marks)*

(c) The line l_2 passes through A and has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ 3 \\ -8 \end{bmatrix}$.

The lines l_1 and l_2 intersect at the point C. Show that the points A, B and C form an isosceles triangle. *(6 marks)*

8 (a) Solve the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{150 \cos 2t}{x}$$

given that $x = 20$ when $t = \frac{\pi}{4}$, giving your solution in the form $x^2 = \mathrm{f}(t)$. *(6 marks)*

(b) The oscillations of a 'baby bouncy cradle' are modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{150 \cos 2t}{x}$$

where x cm is the height of the cradle above its base t seconds after the cradle begins to oscillate.

Given that the cradle is 20 cm above its base at time $t = \frac{\pi}{4}$ seconds, find:

(i) the height of the cradle above its base 13 seconds after it starts oscillating, giving your answer to the nearest millimetre; *(2 marks)*

(ii) the time at which the cradle will first be 11 cm above its base, giving your answer to the nearest tenth of a second. *(2 marks)*

END OF QUESTIONS