

Sec, Cosec, Cot - Exam Question (C3 - Jan '11)

7 (a) Solve the equation $\sec x = -5$, giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. (3 marks)

(b) Show that the equation

$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

can be written in the form

$$\sec^2 x = 25 \quad (4 \text{ marks})$$

(c) Hence, or otherwise, solve the equation

$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. (3 marks)

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7.

a)

$$\sec x = -5 \Rightarrow \cos x = -\frac{1}{5} \Rightarrow x = \cos^{-1}(-0.2) = 1.772 \dots$$

Using the symmetries and period of the graph of $y = \cos x$:

$$x = 2\pi - 1.772 \dots = 4.511 \dots \Rightarrow x = \mathbf{1.77, 4.51 \text{ to 2 d.p.}}$$

b)

$$\frac{\csc x}{1 + \csc x} - \frac{\csc x}{1 - \csc x} = 50$$

$$\csc x \left(\frac{1}{1 + \csc x} - \frac{1}{1 - \csc x} \right) = 50$$

$$\csc x \frac{(1 - \csc x) - (1 + \csc x)}{(1 + \csc x)(1 - \csc x)} = 50$$

$$\csc x \frac{-2 \csc x}{1 - \csc^2 x} = 50$$

$$\frac{-2 \csc^2 x}{1 - (1 + \cot^2 x)} = 50$$

$$\frac{-2 \csc^2 x}{-\cot^2 x} = 50$$

$$2 \left(\frac{1}{\sin^2 x} \right) \left(\frac{\sin^2 x}{\cos^2 x} \right) = 50$$

$$2 \sec^2 x = 50$$

$$\mathbf{\sec^2 x = 25}$$

c)

$$\sec^2 x = 25 \Rightarrow \sec x = \pm 5$$

$$\sec x = -5 \Rightarrow x = 1.77, 4.51 \text{ to 2 d.p.}$$

$$\sec x = 5 \Rightarrow \cos x = \frac{1}{5} \Rightarrow x = 1.37, 4.91 \text{ to 2 d.p.}$$

$$x = \mathbf{1.37, 1.77, 4.51, 4.91 \text{ to 2 d.p.}}$$