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General Certificate of Education (A-level)
January 2011

Mathematics MM2B

(Specification 6360)

Mechanics 2B

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM2B

Q	Solution	Marks	Total	Comments
1(a)	$\mathbf{r} = \int v \, \mathrm{d}t$			
	$= (4t + t^3)\mathbf{i} + (12t - 4t^2)\mathbf{j} + \mathbf{c}$	M1A1		M1 either **i** or **j** term correct. Condone no **c**
	When $t = 0$, $\mathbf{r} = 5\mathbf{i} - 7\mathbf{j}$			
	$\mathbf{c} = 5\mathbf{i} - 7\mathbf{j}$	M1		Any attempt at **c**
	$\mathbf{r} = (5 + 4t + t^3)\mathbf{i} + (-7 + 12t - 4t^2)\mathbf{j}$	A1	4	
(b)	$\mathbf{a} = \frac{\mathrm{d}v}{\mathrm{d}t}$			
	$\mathbf{a} = 6t\,\mathbf{i} - 8\,\mathbf{j}$	M1A1	2	M1 either term correct
(c)	Using $\mathbf{F} = m\mathbf{a}$	M1		Or: using $\mathbf{F} = m\mathbf{a}$
	$\mathbf{F} = 2(6t\,\mathbf{i} - 8\,\mathbf{j})$			$\mathbf{F} = 2(6t\,\mathbf{i} - 8\,\mathbf{j})$
	$= 12t\,\mathbf{i} - 16\,\mathbf{j}$	A1		When $t = 1$, $\mathbf{F} = 12\,\mathbf{i} - 16\,\mathbf{j}$
	$\therefore$ Magnitude of force is			
	$(144t^2 + 256)^{\frac{1}{2}}$ when $t = 1$	M1		Magnitude of force is $(144 + 256)^{\frac{1}{2}}$
	$= 20$ N	A1	4	$= 20$ N
	Total		**10**	
2(a)	PE lost is			
	$= 4 \times g \times 5 \cos 70$	M1A1	2	M1 $4 \times g \times 5 \times \cos$ or $\sin 20$ or 70
	$= 67.0$ J			
(b)	KE is loss of PE $\Rightarrow$ KE is 67.0 J	B1	1	ft
(c)	Using KE $= \frac{1}{2}mv^2$			
	$v^2 = 33.5$	M1		
	Speed of particle is 5.79 m s^{-1}	A1	2	(ft from (b))
	Total		**5**	
3(a)	PE is $400 \times g \times 8$			
	$= 3200\,g$ [or 31 360]	B1	1	
(b)	KE is $\frac{1}{2} \times 400 \times 2^2$			
	$= 800$	B1	1	
(c)	Work done per minute is 32 160 J			
	Power $= 32\,160 \div 60$	M1		$[(a) + (b)] \div 60$
	$= 536$ W	A1	2	CAO Accept 537 from 31 400 in (a)
	Total		**4**	

MM2B(cont)

Q	Solution	Marks	Total	Comments
4(a)	Moments about line AD: $5 \times 30 + 4 \times 10 = 9 \times \bar{x}$ $\bar{x} = \frac{190}{9}$ $= 21.1$ cm	M1A1 A1	 3	M1 2 of 3 terms correct
(b)	Moments about line AB: $5 \times 15 + 4 \times 25 = 9 \times \bar{y}$ $\bar{y} = \frac{175}{9}$ $\bar{y} = 19.4$ cm	 M1A1 A1	 3	 M1 2 of 3 terms correct If moments about DC; 10.6 found SC2
(c)	$\tan\theta = \frac{80}{175}$ or $\frac{8.9}{19.4}$ $= 0.4571$ Angle is $\tan^{-1} 0.4571$ $= 24.6°$	M1 A1 A1 A1	 4	M1 use of tan A1 use of 8.9 or 80 $\left(30-(a)\right)$ Or 0.45876 $65.4^\circ \Rightarrow$ M1A1 only
(d)	Moments about the line PR: (or AD or BC) $30m = 4 \times 20$ or $9 \times \frac{80}{9}$ $m = \frac{8}{3}$	M1 A1 A1	 3	
(e)	Centre of mass is at middle of lamina	E1	1	
	Total		**14**	

MM2B(cont)

Q	Solution	Marks	Total	Comments
5(a)	Resolve vertically $R = mg$ If the particle is on the point of sliding, $F = \mu R$	M1		Ignore all inequalities
	$\therefore F = 0.3R = 0.3mg$	A1		
	Resolving radially: $F = m\omega^2 r$ $0.3mg = m\omega^2 \times 0.8$ $\omega^2 = \dfrac{0.3\times g}{0.8}$	M1		
	$\omega = 1.92$	A1	4	
(b)(i)	45 revolutions per minute = $\dfrac{90\pi}{60}$	M1		
	$= \dfrac{3\pi}{2}$ or 4.71 radians per second	A1	2	
(ii)	Resolving radially: $F = m\omega^2 r$			
	$m\mu g = m\left(\dfrac{3\pi}{2}\right)^2 \times 0.15$	M1A1 A1		M1A1 either side correct A1 second side correct
	$\mu = \dfrac{\left(\dfrac{3\pi}{2}\right)^2 \times 0.15}{g}$			
	$\mu = 0.340$	A1	4	CAO (accept 0.339)
	Total		**10**	
6(a)	By conservation of energy			
	$\dfrac{1}{2}m(5v)^2 = \dfrac{1}{2}m(3v)^2 + mg2a$	M1 A1		M1 for 3 terms , 2 KE and PE
	$8v^2 = 2ag$	A1		
	$v = \sqrt{\dfrac{ag}{4}}$ or $\dfrac{1}{2}\sqrt{ag}$	A1	4	
(b)	Greatest and least values of tension are at the highest and lowest points of its path			
	At top, $T = \dfrac{m(3v)^2}{a} - mg$	M1		
	$= \dfrac{5}{4}mg$	A1ft		ft - must be positive tension
	At B, $T = \dfrac{m(5v)^2}{a} + mg$	M1		
	$= \dfrac{29}{4}mg$	A1ft		
	Ratio is 29 : 5	A1	5	CAO Condone 5 : 29 or 1 : 5.8
	Total		**9**	

MM2B(cont)

Q	Solution	Marks	Total	Comments
7(a)	Work done = $\int_0^e \frac{\lambda x}{l}\,dx$	M1		Condone lack of limits and '*dx*'
	$= \left[\frac{\lambda x^2}{2l}\right]_0^e$	A1		Must include limits from integral
	$= \frac{\lambda e^2}{2l}$	A1	3	AG
(b)(i)	Using $T = \frac{\lambda x}{l}$, $7g = \frac{196x}{2}$	M1		M1 could use *3g* or *4g* – at least 1 side correct
	$x = \frac{14g}{196}$	A1		
	$= 0.7$	A1	3	
(ii)	By C of Energy, when next at rest, EPE (initial) = PE change (for platform) + EPE (when at rest)			
	$\frac{196\times0.7^2}{2\times2} = 4\times g\times(0.7-x)+\frac{196x^2}{2\times2}$	M1A1 A1		M1 3 terms (not including $\frac{1}{2}mv^2$) A1 2 of 3 terms correct A1 all correct
	$2.45 = 2.8 - 4x + 5x^2$ $100x^2 - 80x + 7 = 0$ $(10x - 7)(10x - 1) = 0$ $x = 0.1$	 m1 A1 A1	 6	 [last A1, must give 0.1, not 0.1 and 0.7]
	<u>Alternative</u>			
(b)(ii)	$\frac{196\times0.7^2}{2\times2} = 4gX + \frac{196(0.7-X)^2}{2\times2}$	(M1) (A1) (A1)		(where X is distance moved upwards)
	$4gX = 98\times0.7X + 49X^2$	(m1)		
	$X = 0,\ 0.6$	(A1A1)		
(iii)	Max speed when $T = mg$	M1		
	$4g = \frac{196x}{2}$	A1		
	$x = 0.4$	A1	3	Or mid-point of values 0.2 and 0.6 above SC2
	Total		**15**	

MM2B

Q	Solution	Marks	Total	Comments
8(a)(i)	$F = 65g - 260v$ $= 65(9.8 - 4v)$	B1	1	Accept $260v - 65g$ AG must see $65g$ or 260
(ii)	Using $F = ma$			
	$65\dfrac{\mathrm{d}v}{\mathrm{d}t} = 65(9.8 - 4v)$	M1		Need to see terms in m (condone $-$ sign)
	$\dfrac{\mathrm{d}v}{\mathrm{d}t} = -4(v - 2.45)$	A1	2	AG
(b)	$\dfrac{1}{v - 2.45}\dfrac{\mathrm{d}v}{\mathrm{d}t} = -4$	B1		
	$\int \dfrac{1}{v - 2.45}\,\mathrm{d}v = -\int 4\,\mathrm{d}t$			
	$\ln(v - 2.45) = -4t + c$	M1 A1		M1 log side correct $-4t + c$
	$v - 2.45 = C\mathrm{e}^{-4t}$ $t = 0,\ v = 19.6$ $\therefore C = 17.15$ or $e^{2.84}$	A1		Or $c = \ln 17.15$ or 2.84
	$\therefore v = 2.45 + 17.15\mathrm{e}^{-4t}$ $2.45 + 17.2e^{-4t}$	A1	5	
	Total		**8**	
	TOTAL		**75**	