

Practice examination questions for surds

1. Jan 2005

- 5 (a) Simplify $(\sqrt{12} + 2)(\sqrt{12} - 2)$. *(2 marks)*
- (b) Express $\sqrt{12}$ in the form $m\sqrt{3}$, where m is an integer. *(1 mark)*
- (c) Express $\frac{\sqrt{12} + 2}{\sqrt{12} - 2}$ in the form $a + b\sqrt{3}$, where a and b are integers. *(4 marks)*

2. June 2005

- 5 Express each of the following in the form $m + n\sqrt{3}$, where m and n are integers:
- (a) $(\sqrt{3} + 1)^2$; *(2 marks)*
- (b) $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$. *(3 marks)*

3. Jan 2006

- 1 (a) Simplify $(\sqrt{5} + 2)(\sqrt{5} - 2)$. *(2 marks)*
- (b) Express $\sqrt{8} + \sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer. *(2 marks)*

4. June 2006

- 4 (a) Express $(4\sqrt{5} - 1)(\sqrt{5} + 3)$ in the form $p + q\sqrt{5}$, where p and q are integers. *(3 marks)*
- (b) Show that $\frac{\sqrt{75} - \sqrt{27}}{\sqrt{3}}$ is an integer and find its value. *(3 marks)*

5. Jan 2007

3 (a) Express $\frac{\sqrt{5}+3}{\sqrt{5}-2}$ in the form $p\sqrt{5}+q$, where p and q are integers. (4 marks)

(b) (i) Express $\sqrt{45}$ in the form $n\sqrt{5}$, where n is an integer. (1 mark)

(ii) Solve the equation

$$x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$$

giving your answer in its simplest form. (3 marks)

6. June 2007

2 (a) Express $\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}}$ in the form $n\sqrt{7}$, where n is an integer. (3 marks)

(b) Express $\frac{\sqrt{7}+1}{\sqrt{7}-2}$ in the form $p\sqrt{7}+q$, where p and q are integers. (4 marks)

7. Jan 2008

3 (a) Express $5\sqrt{8} + \frac{6}{\sqrt{2}}$ in the form $n\sqrt{2}$, where n is an integer. (3 marks)

(b) Express $\frac{\sqrt{2}+2}{3\sqrt{2}-4}$ in the form $c\sqrt{2}+d$, where c and d are integers. (4 marks)

8. June 2008

2 It is given that $x = \sqrt{3}$ and $y = \sqrt{12}$.

Find, in the simplest form, the value of:

(a) xy ; (1 mark)

(b) $\frac{y}{x}$; (2 marks)

(c) $(x+y)^2$. (3 marks)

9. Jan 2009

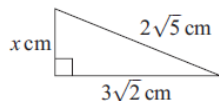
3 (a) Express $\frac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $m+n\sqrt{5}$, where m and n are integers. (4 marks)

(b) Express $\sqrt{45} + \frac{20}{\sqrt{5}}$ in the form $k\sqrt{5}$, where k is an integer. (3 marks)

10.June 2009

2 (a) Express $\frac{5 + \sqrt{7}}{3 - \sqrt{7}}$ in the form $m + n\sqrt{7}$, where m and n are integers. (4 marks)

(b) The diagram shows a right-angled triangle.



The hypotenuse has length $2\sqrt{5}$ cm. The other two sides have lengths $3\sqrt{2}$ cm and x cm. Find the value of x . (3 marks)

11.Jan 2010

4 (a) Show that $\frac{\sqrt{50} + \sqrt{18}}{\sqrt{8}}$ is an integer and find its value. (3 marks)

(b) Express $\frac{2\sqrt{7} - 1}{2\sqrt{7} + 5}$ in the form $m + n\sqrt{7}$, where m and n are integers. (4 marks)

12.June 2010

2 (a) Express $(3 - \sqrt{5})^2$ in the form $m + n\sqrt{5}$, where m and n are integers. (2 marks)

(b) Hence express $\frac{(3 - \sqrt{5})^2}{1 + \sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers. (4 marks)

13.Jan 2011

2 (a) Simplify $(3\sqrt{3})^2$. (1 mark)

(b) Express $\frac{4\sqrt{3} + 3\sqrt{7}}{3\sqrt{3} + \sqrt{7}}$ in the form $\frac{m + \sqrt{21}}{n}$, where m and n are integers. (4 marks)

14.June 2011

2 (a) (i)	Express $\sqrt{48}$ in the form $k\sqrt{3}$, where k is an integer.	<i>(1 mark)</i>
(ii)	Simplify $\frac{\sqrt{48} + 2\sqrt{27}}{\sqrt{12}}$, giving your answer as an integer.	<i>(3 marks)</i>
(b)	Express $\frac{1 - 5\sqrt{5}}{3 + \sqrt{5}}$ in the form $m + n\sqrt{5}$, where m and n are integers.	<i>(4 marks)</i>

15.Jan 2012

3 (a) (i)	Simplify $(3\sqrt{2})^2$.	<i>(1 mark)</i>
(ii)	Show that $(3\sqrt{2} - 1)^2 + (3 + \sqrt{2})^2$ is an integer and find its value.	<i>(4 marks)</i>
(b)	Express $\frac{4\sqrt{5} - 7\sqrt{2}}{2\sqrt{5} + \sqrt{2}}$ in the form $m - \sqrt{n}$, where m and n are integers.	<i>(4 marks)</i>

16.June 2012

1	Express $\frac{5\sqrt{3} - 6}{2\sqrt{3} + 3}$ in the form $m + n\sqrt{3}$, where m and n are integers.	<i>(4 marks)</i>
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17.Jan 2013

3 (a) (i)	Express $\sqrt{18}$ in the form $k\sqrt{2}$, where k is an integer.	<i>(1 mark)</i>
(ii)	Simplify $\frac{\sqrt{8}}{\sqrt{18} + \sqrt{32}}$.	<i>(3 marks)</i>
(b)	Express $\frac{7\sqrt{2} - \sqrt{3}}{2\sqrt{2} - \sqrt{3}}$ in the form $m + \sqrt{n}$, where m and n are integers.	<i>(4 marks)</i>

Answers

1. Jan 2005

5(a)	$(\sqrt{12})^2 - 2^2$ attempt to multiply out (= $12 - 4$) = 8	M1	2	May have $\sqrt{12}$ terms
		A1		
(b)	$2\sqrt{3}$	B1	1	
(c)	Multiplying top and bottom by $\sqrt{12} + 2$ Numerator = $12 + 4\sqrt{12} + 4$ Expression = $\frac{16 + 4\sqrt{12}}{8}$ or $\frac{16 + 8\sqrt{3}}{8}$ = $2 + \sqrt{3}$	B1	4	Or $\sqrt{3} + 1$ etc At least 3 terms multiplied out on top OE in $\sqrt{3}$ fit denominator from (a); or correct but numerator correct (unsimplified)
		M1		
		A1✓		
Total			7	

2. June 2005

5(a)	$3 + 1 + 2\sqrt{3}$ = $4 + 2\sqrt{3}$	M1	2	Multiplied out At least 3 terms with $\sqrt{3}$ term $m = 4, n = 2$
		A1		
(b)	Multiplying top and bottom by $\sqrt{3} + 1$ Denominator = $3 - 1 = 2$ Expression = $\frac{4 + 2\sqrt{3}}{2}$ = $2 + \sqrt{3}$	M1	3	CSO $m = 2, n = 1$
		B1		
		A1		
Total			5	

3. Jan 2006

Q	Solution	Marks	Total	Comments
1(a)	$(\sqrt{5})^2 + 2\sqrt{5} - 2\sqrt{5} - 4 = 1$	M1	2	Multiplying out or difference of two squares attempted Full marks for correct answer /no working
		A1		
(b)	$\sqrt{8} = 2\sqrt{2}$; $\sqrt{18} = 3\sqrt{2}$ Answer = $5\sqrt{2}$	M1	2	Either correct Full marks for correct answer /no working
		A1		
Total			4	

4. June 2006

Q	Solution	Marks	Total	Comments
4(a)	$4(\sqrt{5})^2 + 12\sqrt{5} - \sqrt{5} - 3$ $4(\sqrt{5})^2 = 4 \times 5$ (= 20) Answer = $17 + 11\sqrt{5}$	M1	3	Multiplied out At least 3 terms with $\sqrt{5}$ term
		B1		
		A1		
(b)	Either $\sqrt{75} = \sqrt{25}\sqrt{3}$ or $\sqrt{27} = \sqrt{9}\sqrt{3}$ Expression = $\frac{5\sqrt{3} - 3\sqrt{3}}{\sqrt{3}}$ = 2	M1	3	Or multiplying top and bottom by $\sqrt{3}$ or $\frac{\sqrt{225} - \sqrt{81}}{3}$ or $\sqrt{25} - \sqrt{9}$ or $5 - 3$ CSO
		A1		
		A1		
Total			6	

5.Jan 2007

Q	Solution	Marks	Total	Comments
3(a)	$\frac{\sqrt{5+3} \times \sqrt{5+2}}{\sqrt{5-2} \times \sqrt{5+2}}$	M1	4	Multiplying top & bottom by $\pm(\sqrt{5+2})$
	Numerator = $5+3\sqrt{5}+2\sqrt{5}+6$	M1		Multiplying out (condone one slip)
	= $5\sqrt{5}+11$	A1		$\pm(\sqrt{5+3})(\sqrt{5+2})$
	Final answer = $5\sqrt{5}+11$	A1		With clear evidence that denominator =1
(b)(i)	$\sqrt{45} = 3\sqrt{5}$	B1	1	
(ii)	$\sqrt{20} = \sqrt{4}\sqrt{5}$ or $4\sqrt{5} = \sqrt{4} \times \sqrt{20}$ or attempt to have equation with $\sqrt{5}$ or $\sqrt{20}$ only	M1	3	Both sides
	$[x \ 2\sqrt{5} = 7\sqrt{5} - 3\sqrt{5}]$ or $x\sqrt{20} = 2\sqrt{20}$	A1		or $x = \sqrt{4}$
	$x = 2$	A1		CSO
Total			8	

6.June 2007

2(a)	$\frac{\sqrt{63}}{3} = \sqrt{7}$ or $\frac{3\sqrt{7}}{3}$	B1	3	$\frac{(\sqrt{7}\sqrt{63} + 14 \times 3)}{3\sqrt{7}}$
	$\frac{14}{\sqrt{7}} = 2\sqrt{7}$ or $\frac{14\sqrt{7}}{7}$	B1		or $\frac{\sqrt{7}}{\sqrt{7}} ()$ M1
	$\Rightarrow \text{sum} = 3\sqrt{7}$	B1		\Rightarrow correct answer with all working correct A2
(b)	Multiply by $\frac{\sqrt{7}+2}{\sqrt{7}+2}$	M1	4	multiplied out (allow one slip) $9+3\sqrt{7}$
	Denominator = $7 - 4 = 3$	A1		
	Numerator = $(\sqrt{7})^2 + \sqrt{7} + 2\sqrt{7} + 2$	m1		
	Answer = $\sqrt{7} + 3$	A1		
Total			7	

7.Jan 2008

Q	Solution	Marks	Total	Comments
3(a)	$5\sqrt{8} = 10\sqrt{2}$	B1	3	Or $\frac{5\sqrt{16}+6}{\sqrt{2}}$ gets B1
	$\frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} (=3\sqrt{2})$	M1		then M1 for rationalising; and A1 answer
	Answer = $13\sqrt{2}$	A1		$n = 13$
(b)	$\frac{\sqrt{2}+2}{3\sqrt{2}-4} \times \frac{3\sqrt{2}+4}{3\sqrt{2}+4}$	M1	4	Multiplying top & bottom by $\pm(3\sqrt{2}+4)$
	Numerator = $6+6\sqrt{2}+4\sqrt{2}+8$	m1		Multiplying out (condone one slip)
	Denominator = $18-16 (=2)$	B1		
	Final answer = $5\sqrt{2}+7$	A1		
Total			7	

8.June 2008

2(a)	$xy = 6$	B1	1	B0 for $\sqrt{36}$ or ± 6
(b)	$\frac{y}{x} = \frac{2\sqrt{3}}{\sqrt{3}}$ or $\sqrt{\frac{12}{3}}$ or $\sqrt{\frac{4}{1}}$ or $\frac{\sqrt{12}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= 2$	M1 A1	2	Allow M1 for ± 2
(c)	$x^2 + 2xy + y^2$ or $(\sqrt{3} + 2\sqrt{3})^2$ correct Correct with 2 of $x^2, y^2, 2xy$ simplified $3 + 2\sqrt{36} + 12$ or $3^2 \times 3$ or $(3\sqrt{3})^2$ $= 27$	M1 A1 A1	3	or $(\sqrt{3} + \sqrt{12})(\sqrt{3} + \sqrt{12})$ expanded as 4 terms – no more than one slip Correct but unsimplified – one more step
Total			6	

9.Jan 2009

3(a)	$\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ Numerator = $21 + 3\sqrt{5} - 7\sqrt{5} - (\sqrt{5})^2$ Denominator = $9 - 5 = 4$ $Answer = 4 - \sqrt{5}$	M1 m1 B1 A1	4	Multiply by $\frac{3-\sqrt{5}}{3-\sqrt{5}}$ or $\frac{\sqrt{5}-3}{\sqrt{5}-3}$ Condone one slip $16 - 4\sqrt{5}$ (Or $5 - 9 = -4$ from other conjugate) CSO
(b)	$\sqrt{45} = 3\sqrt{5}$ $\frac{20}{\sqrt{5}} = \frac{20\sqrt{5}}{5}$ Sum = $7\sqrt{5}$	B1 M1 A1	3	May score if combined as one expression Must have 5 in denominator
Total			7	

10.June 2009

2(a)	$\frac{5+\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}}$ Numerator = $15 + 5\sqrt{7} + 3\sqrt{7} + 7$ Denominator = $9 - 7 (= 2)$ (Answer =) $11 + 4\sqrt{7}$	M1 m1 B1 A1	4	Condone one error or omission Must be seen as the denominator
(b)	$(2\sqrt{5})^2 = 20$ or $(3\sqrt{2})^2 = 18$ their $(2\sqrt{5})^2 - (3\sqrt{2})^2$ $(x^2 = 20 - 18)$ $(\Rightarrow x =) \sqrt{2}$	B1 M1 A1	3	Either correct Condone missing brackets and x^2 $x^2 = 2 \Rightarrow$ B1, M1 $\pm\sqrt{2}$ scores A0 Answer only of 2 scores B0, M0 Answer only of $\sqrt{2}$ scores 3 marks
Total			7	

11.Jan 2010

4(a)	$\sqrt{50}=5\sqrt{2}; \sqrt{18}=3\sqrt{2}; \sqrt{8}=2\sqrt{2}$ At least two of these correct	M1		or $\times \frac{\sqrt{8}}{\sqrt{8}}$ or $\left(\times \frac{\sqrt{2}}{\sqrt{2}}\right)$ or $\sqrt{\frac{25}{4}} + \sqrt{\frac{9}{4}}$
	$\frac{5\sqrt{2}+3\sqrt{2}}{2\sqrt{2}}$ <i>Answer</i> = 4	A1		any correct expression all in terms of $\sqrt{2}$ or with denominator of 8, 4 or 2 simplifying numerator eg $\frac{\sqrt{400} + \sqrt{144}}{8}$
(b)	$\frac{(2\sqrt{7}-1)(2\sqrt{7}-5)}{(2\sqrt{7}+5)(2\sqrt{7}-5)}$	M1		OE
	<i>numerator</i> = $4 \times 7 - 2\sqrt{7} - 10\sqrt{7} + 5$ <i>denominator</i> = 3 <i>Answer</i> = $11 - 4\sqrt{7}$	m1 B1 A1	3	CSO (condone one error or omission) (seen as denominator)
Total			7	

12.June 2010

2(a)	$(3-\sqrt{5})^2 = 9 - 6\sqrt{5} + (\sqrt{5})^2$ $= 14 - 6\sqrt{5}$	M1 A1	2	Allow one slip in one of these terms M0 if middle term is omitted
	(b) $\frac{(3-\sqrt{5})^2}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$ $14 + 6\sqrt{5}\sqrt{5} - 6\sqrt{5} - 14\sqrt{5}$ $(= 44 - 20\sqrt{5})$ (Denominator) = -4 (Answer) = $-11 + 5\sqrt{5}$	M1 m1 B1 A1	4	or $\dots \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$ Expanding <i>their</i> numerator (condone one error or omission) Must be seen as denominator Accept "answer = $5\sqrt{5} - 11$ "
Total			6	

13.Jan 2011

Q	Solution	Marks	Total	Comments
2(a)	27	B1	1	
(b)	$\frac{4\sqrt{3}+3\sqrt{7}}{3\sqrt{3}+\sqrt{7}} \times \frac{3\sqrt{3}-\sqrt{7}}{3\sqrt{3}-\sqrt{7}}$	M1		
	(Numerator =) $36 + 9\sqrt{21} - 4\sqrt{21} - 21$	m1		expanding numerator condone one slip or omission
	(Denominator =) 20 $\frac{15+5\sqrt{21}}{20}$	B1		must be seen as denominator
	$= \frac{3+\sqrt{21}}{4}$	A1cso	4	$m=3, n=4$ condone $\frac{3}{4} + \frac{\sqrt{21}}{4}$
Total			5	

14.June 2011

Q	Solution	Marks	Total	Comments
2(a)(i)	$\sqrt{48} = 4\sqrt{3}$	B1	1	condone $k = 4$ stated
(ii)	$\frac{4\sqrt{3} + 6\sqrt{3}}{2\sqrt{3}}$	M1		attempt to write each term in form $k\sqrt{3}$ with at least 2 terms correctly obtained
		A1		correct unsimplified in terms of $\sqrt{3}$ only
	= 5	A1cso	3	must simplify fraction to 5
				Alternative 1 $\times \frac{\sqrt{12}}{\sqrt{12}}$ (or $\times \frac{\sqrt{3}}{\sqrt{3}}$) M1
				correct with integer terms = $\frac{24+36}{12}$ A1
				= 5 A1cso
				Alternative 2 $\frac{\sqrt{48} + \sqrt{108}}{\sqrt{12}}$ M1
				= $\sqrt{4} + \sqrt{9}$ A1
				= 5 A1cso
				Alternative 3 $\sqrt{\frac{48}{12}} + 2\sqrt{\frac{27}{12}}$ M1
				= $2 + 2\sqrt{\frac{9}{4}}$ A1
				= 5 A1cso
				if hybrid of methods used, award M1 and most appropriate first A1
				NMS (answer =) 5 scores full marks
(b)	$\frac{1-5\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$	M1		
	(numerator =) $3 - \sqrt{5} - 15\sqrt{5} + 25$	m1		correct unsimplified but must write $5\sqrt{5}\sqrt{5} = 25$ PI by 28 seen later
	(denominator = $9 - 5$) = 4	B1		must be seen as denominator
	giving $\frac{28 - 16\sqrt{5}}{4}$			
	(answer =) $7 - 4\sqrt{5}$	A1	4	$m = 7, n = -4$
	Total		8	

15.Jan 2012

Q	Solution	Marks	Total	Comments
3(a)(i)	$(3\sqrt{2})^2 = 18$	B1	1	
(ii)	$(3\sqrt{2} - 1)^2 = \text{'their } 18' - 3\sqrt{2} - 3\sqrt{2} + 1$	M1		FT their $(3\sqrt{2})^2$
	= $18 - 3\sqrt{2} - 3\sqrt{2} + 1$	A1		(= $19 - 6\sqrt{2}$)
	$(3 + \sqrt{2})^2 = 9 + 3\sqrt{2} + 3\sqrt{2} + 2$	B1		(= $11 + 6\sqrt{2}$)
	\Rightarrow Sum = 30	A1cso	4	

(b)	$\frac{4\sqrt{5}-7\sqrt{2}}{2\sqrt{5}+\sqrt{2}} \times \frac{2\sqrt{5}-\sqrt{2}}{2\sqrt{5}-\sqrt{2}}$	M1		
	Numerator = $8(\sqrt{5})^2 - 4\sqrt{5}\sqrt{2} - 14\sqrt{5}\sqrt{2} + 7(\sqrt{2})^2$	m1		correct unsimplified $(= 54 - 18\sqrt{10})$
	Denominator = $(2\sqrt{5})^2 - (\sqrt{2})^2$ $= 18$	B1		must be seen as denominator
	\Rightarrow Answer = $3 - \sqrt{10}$	Alcso	4	
Total			9	

16.June 2012

Q	Solution	Marks	Total	Comments
1	$\frac{5\sqrt{3}-6}{2\sqrt{3}+3} \times \frac{2\sqrt{3}-3}{2\sqrt{3}-3}$	M1		
	(Numerator =) $30 - 15\sqrt{3} - 12\sqrt{3} + 18$	m1		correct $(= 48 - 27\sqrt{3})$
	(Denominator = $12 - 9 =$) 3	B1		must be seen as denominator
	$\left(\frac{48 - 27\sqrt{3}}{3}\right) = 16 - 9\sqrt{3}$	A1	4	CSO; accept $16 + -9\sqrt{3}$
Total			4	

17.Jan 2013

Q	Solution	Marks	Total	Comments
3(a)(i)	$\sqrt{18} = 3\sqrt{2}$	B1	1	Condone $k = 3$
(ii)	$\frac{2\sqrt{2}}{3\sqrt{2}+4\sqrt{2}}$	M1		attempt to write each term in form $n\sqrt{2}$ with at least 2 terms correct
	$= \frac{2}{7}$	A1		correct unsimplified
		A1	3	or $\times \frac{\sqrt{2}}{\sqrt{2}}$ M1
				integer terms = $\frac{4}{6+8}$ A1
				$= \frac{2}{7}$ A1
(b)	$\frac{7\sqrt{2}-\sqrt{3}}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}}$	M1		
	(numerator =) $14 \times 2 - 2\sqrt{6} + 7\sqrt{6} - 3$	m1		correct unsimplified but must simplify $(\sqrt{2})^2$, $(\sqrt{3})^2$ and $\sqrt{2} \times \sqrt{3}$ correctly
	(denominator = $8 - 3 =$) 5	B1		must be seen or identified as denominator
	(Answer =) $5 + \sqrt{6}$	Alcso	4	giving $\frac{25+5\sqrt{6}}{5}$ $m = 5, n = 6$
Total			8	