AQA MPC1 Core 1 Mathematics 19 May 2014

Question Paper and Worked Solutions

Please note, this document represents my own solutions to the questions, is entirely unofficial and is not related to the mark scheme (which I have not seen). Therefore, while it should help you see how to do the questions, it won't include every valid method or give you a break down of the mark allocation. If you spot any errors, or think you have found a better solution, please email me so I can update it.

1 The point A has coordinates $(-1, 2)$ and the point B has coordinates $(3, -5)$.

(a) (i) Find the gradient of AB.

[2 marks]

(ii) Hence find an equation of the line AB, giving your answer in the form $px + qy = r$, where p, q and r are integers.

[3 marks]

(b) The midpoint of AB is M.

(i) Find the coordinates of M.

[1 mark]

(ii) Find an equation of the line which passes through M and which is perpendicular to AB.

[3 marks]

(c) The point C has coordinates $(k, 2k+3)$. Given that the distance from A to C is $\sqrt{13}$, find the two possible values of the constant k.

[4 marks]

1.
a)
i.

$$Gradient = \frac{y - step}{x - step} = \frac{-5-2}{3--1} = -\frac{\mathbf{7}}{\mathbf{4}} \quad or \quad -\mathbf{1.75}$$

ii.

$$y - y_1 = m(x - x_1) \quad \Rightarrow \quad y - 2 = -\frac{7}{4}(x+1) \quad \Rightarrow \quad 4y - 8 = -7x - 7 \quad \Rightarrow \quad \mathbf{7x + 4y = 1}$$

b)
i.

$$Midpoint: \left(\frac{-1+3}{2}, \frac{2 \pm 5}{2}\right) = \left(1, -\frac{3}{2}\right)$$

ii.

$$Perpendicular \quad \Rightarrow \quad m = -\frac{1}{-\frac{7}{4}} = \frac{4}{7}$$

$$y - y_1 = m(x - x_1) \quad \Rightarrow \quad y - 1 = \frac{4}{7}\left(x - -\frac{3}{2}\right) \quad \Rightarrow \quad 7y - 7 = 4x + 6 \quad \Rightarrow \quad \mathbf{4x - 7y = -13}$$

c)

$$Distance = \sqrt{(k - -1)^2 + (2k + 3 - 2)^2} = \sqrt{13} \quad \Rightarrow \quad (k+1)^2 + (2k+1)^2 = 13$$

$$\Rightarrow \quad k^2 + 2k + 1 + 4k^2 + 4k + 1 = 13 \quad \Rightarrow \quad 5k^2 + 6k - 11 = 0 \quad \Rightarrow \quad (5k + 11)(k - 1) = 0$$

$$\Rightarrow \quad \boldsymbol{k = -\frac{11}{5} = -2.2 \quad or \quad k = 1}$$

2 A rectangle has length $(9+5\sqrt{3})$ cm and area $(15+7\sqrt{3})$ cm^2.

Find the width of the rectangle, giving your answer in the form $(m+n\sqrt{3})$ cm, where m and n are integers.

[4 marks]

2.

$$Width = \frac{Area}{Length} = \frac{15+7\sqrt{3}}{9+5\sqrt{3}} = \frac{(15+7\sqrt{3})(9-5\sqrt{3})}{(9+5\sqrt{3})(9-5\sqrt{3})} = \frac{135+63\sqrt{3}-75\sqrt{3}-105}{81-75} = \frac{30-12\sqrt{3}}{6} = \mathbf{5-2\sqrt{3}}$$

$$(\boldsymbol{m=5 \quad and \quad n=-2})$$

3 A curve has equation $y = 2x^5 + 5x^4 - 1$.

(a) Find:

(i) $\frac{dy}{dx}$

[2 marks]

(ii) $\frac{d^2y}{dx^2}$

[1 mark]

(b) The point on the curve where $x = -1$ is P.

(i) Determine whether y is increasing or decreasing at P, giving a reason for your answer.

[2 marks]

(ii) Find an equation of the tangent to the curve at P.

[3 marks]

(c) The point $Q(-2, 15)$ also lies on the curve. Verify that Q is a maximum point of the curve.

[4 marks]

3.

a)

i.

$$y = 2x^5 + 5x^4 - 1 \quad \Rightarrow \quad \frac{dy}{dx} = \mathbf{10x^4 + 20x^3}$$

ii.

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \mathbf{40x^3 + 60x^2}$$

b)

i.

$$x = -1 \quad \Rightarrow \quad \frac{dy}{dx} = 10(-1)^4 + 20(-1)^3 = -10 < 0 \quad \Rightarrow \quad \boldsymbol{Decreasing\ as\ gradient\ is\ negative}$$

ii.

$$x = -1 \quad \Rightarrow \quad y = 2(-1)^5 + 5(-1)^4 - 1 = 2 \quad \Rightarrow \quad P: (-1, 2)$$

$$y - y_1 = m(x - x_1) \quad \Rightarrow \quad y - 2 = -10(x - -1) \quad \Rightarrow \quad y - 2 = -10x - 10 \quad \Rightarrow \quad \mathbf{y = -10x - 8}$$

c)

$$x = -2 \quad \Rightarrow \quad \frac{dy}{dx} = 10(-2)^4 + 20(-2)^3 = 160 - 160 = 0 \quad \Rightarrow \quad \boldsymbol{Stationary\ Point}$$

$$x = -2 \quad \Rightarrow \quad \frac{d^2y}{dx^2} = 40(-2)^3 + 60(-2)^2 = -320 + 240 = -80 < 0 \quad \Rightarrow \quad \boldsymbol{Maximum}$$

4 (a) (i) Express $16 - 6x - x^2$ in the form $p - (x+q)^2$ where p and q are integers.

[2 marks]

(ii) Hence write down the maximum value of $16 - 6x - x^2$.

[1 mark]

(b) (i) Factorise $16 - 6x - x^2$.

[1 mark]

(ii) Sketch the curve with equation $y = 16 - 6x - x^2$, stating the values of x where the curve crosses the x-axis and the value of the y-intercept.

[3 marks]

4.

a)

i.

$$16 - 6x - x^2 = -(x^2 + 6x - 16) = -((x+3)^2 - 9 - 16) = -((x+3)^2 - 25) = \mathbf{25 - (x+3)^2}$$

ii.

$$Maximum\ when\ (x+3)^2 = 0 \quad \Rightarrow \quad x = -3 \quad \Rightarrow \quad 25 - (x+3)^2 = 25 - (-3+3)^2 = \mathbf{25}$$

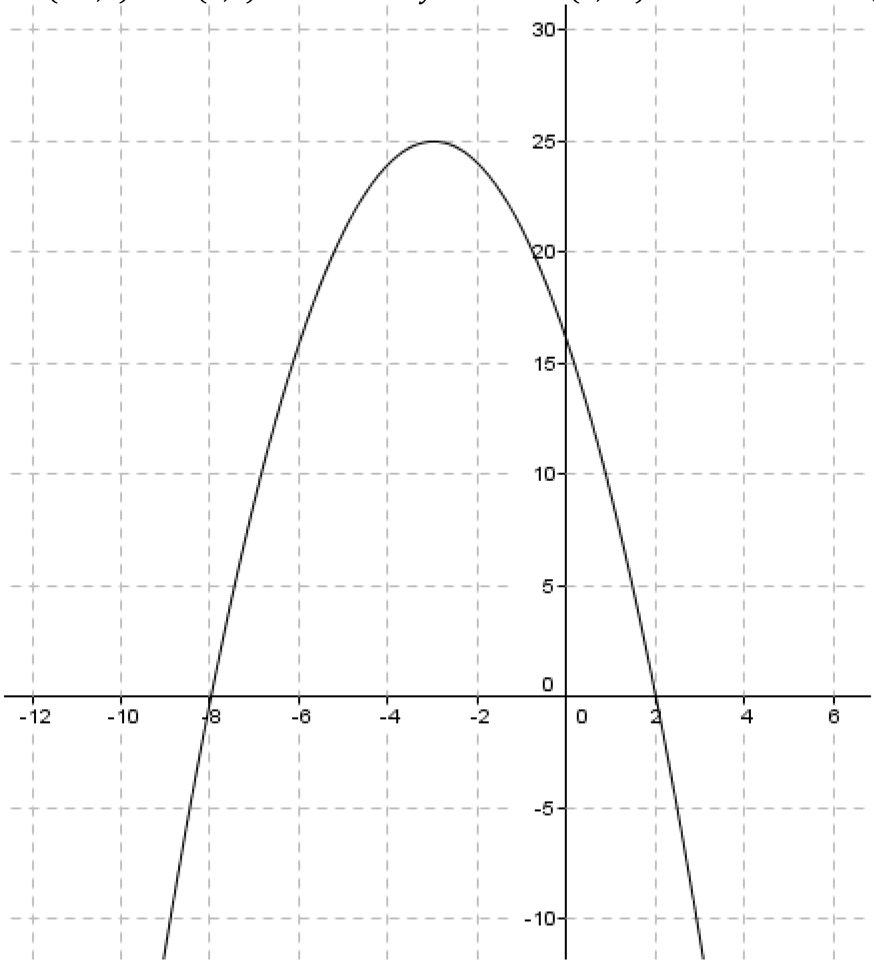
b)

i.

$$16 - 6x - x^2 = -(x^2 + 6x - 16) = -(\boldsymbol{x+8})(\boldsymbol{x-2}) \quad \boldsymbol{or} \quad (\boldsymbol{x+8})(\boldsymbol{2-x})$$

ii.

$$Crosses\ at\ (-8,0)\ and\ (2,0) \qquad Crosses\ y-axis\ at\ (0,16) \qquad Maximum\ at\ (-3,25):$$



5 The polynomial $p(x)$ is given by

$$p(x) = x^3 + cx^2 + dx + 3$$

where c and d are integers.

(a) Given that $x + 3$ is a factor of $p(x)$, show that

$$3c - d = 8$$

[2 marks]

(b) The remainder when $p(x)$ is divided by $x - 2$ is 65.

Obtain a further equation in c and d.

[2 marks]

(c) Use the equations from parts **(a)** and **(b)** to find the value of c and the value of d.

[3 marks]

5.

a)

$$(x+3)\ is\ a\ factor \quad \Leftrightarrow \quad -3\ is\ a\ root \quad \Leftrightarrow \quad p(-3) = 0$$

$$p(-3) = (-3)^3 + c(-3)^2 + d(-3) + 3 = 0 \quad \Rightarrow \quad -27 + 9c - 3d + 3 = 0 \quad \Rightarrow \quad \mathbf{3c - d = 8}$$

b)

$$Remainder\ when\ p(x)\ is\ divided\ by\ (x-2)\ is\ R \quad \Leftrightarrow \quad p(2) = R$$

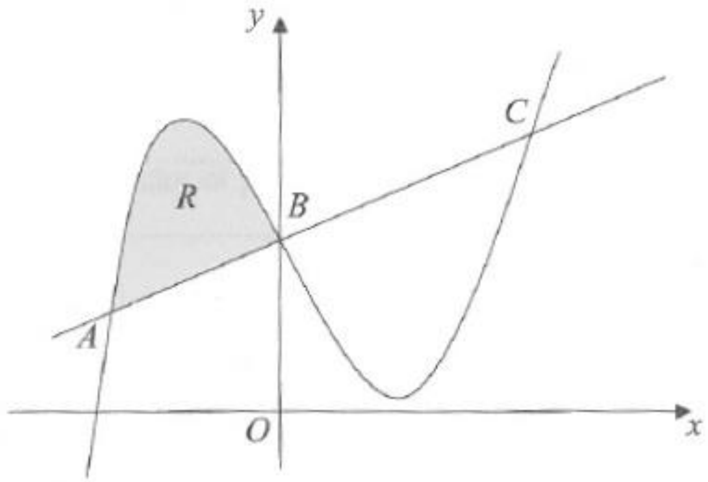
$$p(2) = 2^3 + c(2^2) + d(2) + 3 = 65 \quad \Rightarrow \quad 8 + 4c + 2d + 3 = 65 \quad \Rightarrow \quad \mathbf{2c + d = 27}$$

c)

$$Solving\ simultaneously:$$
$$(3c - d) + (2c + d) = 8 + 27$$

$$5c = 35 \quad \Rightarrow \quad \mathbf{c = 7} \quad \Rightarrow \quad 2(7) + d = 27 \quad \Rightarrow \quad \mathbf{d = 13}$$

6 The diagram shows a curve and a line which intersect at the points A, B and C.



The curve has equation $y = x^3 - x^2 - 5x + 7$ and the straight line has equation $y = x + 7$. The point B has coordinates $(0, 7)$.

(a) (i) Show that the x-coordinates of the points A and C satisfy the equation

$$x^2 - x - 6 = 0$$

[2 marks]

(ii) Find the coordinates of the points A and C.

[3 marks]

(b) Find $\int (x^3 - x^2 - 5x + 7)\, dx$.

[3 marks]

(c) Find the area of the shaded region R bounded by the curve and the line segment AB.

[4 marks]

6.
a)
i.

$$Solving\ simultaneously:\quad x + 7 = x^3 - x^2 - 5x + 7 \quad \Rightarrow \quad x^3 - x^2 - 6x = 0 \quad \Rightarrow \quad x(x^2 - x - 6) = 0$$

$$\Rightarrow \quad x = 0 \quad or \quad x^2 - x - 6 = 0$$

$$\boldsymbol{Since\ B\ is\ the\ point\ where\ x = 0, the\ x\ coordinates\ of\ the\ points\ A\ and\ C\ satisfy:\quad x^2 - x - 6 = 0}$$

ii.

$$x^2 - x - 6 = 0 \quad \Rightarrow \quad (x+2)(x-3) = 0 \quad \Rightarrow \quad x = -2 \ \ or\ x = 3$$

$$Substituting\ into\ y = x + 7:\ \boldsymbol{Point\ A}:\ \boldsymbol{(-2, 5)} \quad \boldsymbol{and} \quad \boldsymbol{Point\ C}:\ \boldsymbol{(3, 10)}$$

b)

$$\int x^3 - x^2 - 5x + 7\, dx = \boldsymbol{\frac{x^4}{4} - \frac{x^3}{3} - \frac{5x^2}{2} + 7x + C}$$

c)

$$Area = \int_{-2}^{0} x^3 - x^2 - 5x + 7\, dx - \left(\frac{a+b}{2}h\right) = \left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{5x^2}{2} + 7x\right]_{-2}^{0} - \left(\frac{5+7}{2}(2)\right)$$

$$= \left\{(0) - \left(\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - \frac{5(-2)^2}{2} + 7(-2)\right)\right\} - (12) = \frac{52}{3} - 12 = \boldsymbol{\frac{16}{3}}$$

7 A circle with centre C has equation $x^2 + y^2 - 10x + 12y + 41 = 0$. The point $A(3, -2)$ lies on the circle.

(a) Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$

[3 marks]

(b) (i) Write down the coordinates of C.

[1 mark]

(ii) Show that the circle has radius $n\sqrt{5}$, where n is an integer.

[2 marks]

(c) Find the equation of the tangent to the circle at the point A, giving your answer in the form $x + py = q$, where p and q are integers.

[5 marks]

(d) The point B lies on the tangent to the circle at A and the length of BC is 6. Find the length of AB.

[3 marks]

7.

a)

$$\textit{Completing the square:}\quad x^2 - 10x + y^2 + 12y + 41 = 0 \quad\Rightarrow\quad (x-5)^2 - 25 + (y+6)^2 - 36 + 41 = 0$$

$$\Rightarrow\quad \mathbf{(x-5)^2 + (y+6)^2 = 20}$$

b)

i.

$$C:\ (\mathbf{5}, \mathbf{-6})$$

ii.

$$r^2 = 20 \quad\Rightarrow\quad r = \sqrt{20} = \sqrt{4\times 5} = \sqrt{4}\sqrt{5} = \mathbf{2\sqrt{5}}$$

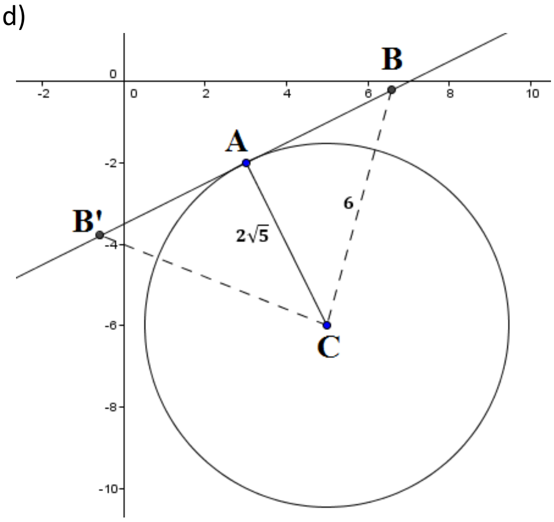
c)

$$\textit{Gradient of normal} = \textit{gradient between } (5,-6) \textit{ and } (3,-2) = \frac{y-step}{x-step} = \frac{4}{-2} = -2$$

$$\textit{Gradient of tangent} = -\frac{1}{\textit{gradient of normal}} = -\frac{1}{-2} = \frac{1}{2}$$

$$y - y_1 = m(x - x_1) \quad\Rightarrow\quad y - -2 = \frac{1}{2}(x-3) \quad\Rightarrow\quad 2y + 4 = x - 3 \quad\Rightarrow\quad \mathbf{x - 2y = 7}$$

d)



B could lie in one of two places, but the distance AB is the same.

$B\hat{A}C = 90°$ since a tangent and a radius always meet at 90°.

$$AC = 2\sqrt{5} \quad and \quad BC = 6 \quad\Rightarrow\quad AB^2 + \left(2\sqrt{5}\right)^2 = 6^2$$

$$\Rightarrow\quad AB^2 = 36 - 20 \quad\Rightarrow\quad AB = \sqrt{16} = \mathbf{4}$$

8 Solve the following inequalities:

(a) $3(1-2x)-5(3x+2)>0$ **[2 marks]**

(b) $6x^2 \leqslant x+12$ **[4 marks]**

8.

a)

$$3(1-2x)-5(3x+2)>0 \quad \Rightarrow \quad 3-6x-15x-10>0 \quad \Rightarrow \quad -7-21x>0$$

$$\Rightarrow \quad -7>21x \quad \Rightarrow \quad -\frac{1}{3}>x \quad \boldsymbol{or} \quad x<-\frac{1}{3}$$

b)

$$6x^2 \le x+12 \quad \Rightarrow \quad 6x^2-x-12 \le 0$$

$$\textit{Critical points occur when } 6x^2-x-12=0 \quad \Rightarrow \quad 6x^2+8x-9x-12=0$$

Alternative 1: Factorising

$$6x^2-x-12=6x^2+8x-9x-12=2x(3x+4)-3(3x+4)=(3x+4)(2x-3)$$

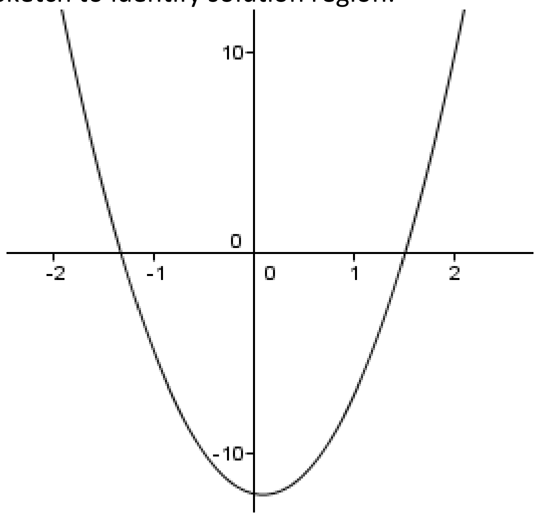
$$(3x+4)(2x-3)=0 \quad \Rightarrow \quad x=-\frac{4}{3} \quad or \quad x=\frac{3}{2}$$

Alternative 2: Using the formula

$$6x^2-x-12=0 \quad \Rightarrow \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{1\pm\sqrt{1-4\times 6\times -12}}{12}=\frac{1\pm\sqrt{289}}{12}$$

$$\Rightarrow \quad x=\frac{1+17}{12}=\frac{3}{2} \quad or \quad x=\frac{1-17}{12}=-\frac{4}{3}$$

Sketch to identify solution region:



Positive quadratic, crossing the x-axis at $\frac{3}{2}$ and $-\frac{4}{3}$.

$$6x^2-x-12 \le 0 \quad \Rightarrow \quad -\frac{4}{3} \le x \le \frac{3}{2}$$