AQA MM1B Mechanics 1 Mathematics 16 June 2014

Question Paper and Worked Solutions

Please note, this document represents my own solutions to the questions, is entirely unofficial and is not related to the mark scheme (which I have not seen). Therefore, while it should help you see how to do the questions, it won't include every valid method or give you a break down of the mark allocation. If you spot any errors, or think you have found a better solution, please email me so I can update it.

1 A car is travelling along a straight horizontal road. It is moving at $14\,\mathrm{m\,s^{-1}}$ when it starts to accelerate. It accelerates at $0.8\,\mathrm{m\,s^{-2}}$ for 12 seconds.

(a) Find the speed of the car at the end of the 12 seconds.

[3 marks]

(b) Find the distance travelled during the 12 seconds.

[3 marks]

(c) The mass of the car is 1400 kg. A horizontal forward driving force of 1600 N acts on the car during the 12 seconds. Find the magnitude of the resistance force that acts on the car.

[3 marks]

1.

a)

$$s = - \quad u = 14 \quad v = - \quad a = 0.8 \quad t = 12$$

$$v = u + at = 14 + 0.8 \times 12 = \mathbf{23.6\ ms^{-1}}$$

b)

$$s = ut + \frac{1}{2}at^2 = 14 \times 12 + \frac{1}{2} \times 0.8 \times 12^2 = \mathbf{225.6\ m}$$

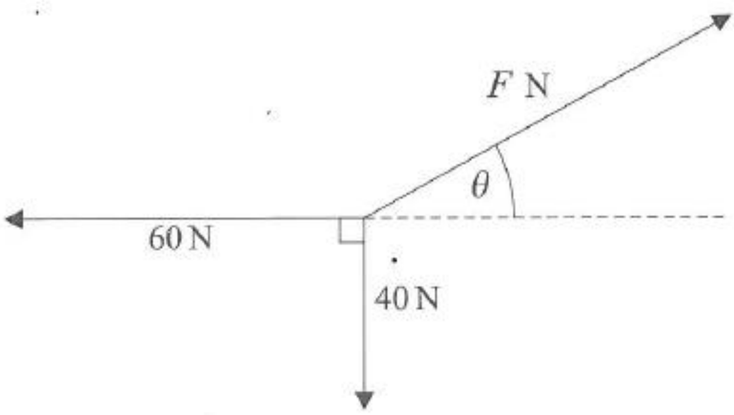
c)

$$F = - \quad m = 1400 \quad a = 0.8$$

$$Resultant\ force: \quad F = ma = 0.8 \times 1400 = 1120 \quad \Longrightarrow \quad Driving\ force - Resistance\ force = 1120$$

$$\Longrightarrow \quad 1600 - Resistance\ force = 1120 \quad \Longrightarrow \quad Resistance\ force = \mathbf{480\ N}$$

2 Three forces are in equilibrium in a vertical plane, as shown in the diagram. There is a vertical force of magnitude 40 N and a horizontal force of magnitude 60 N. The third force has magnitude F newtons and acts at an angle θ above the horizontal.



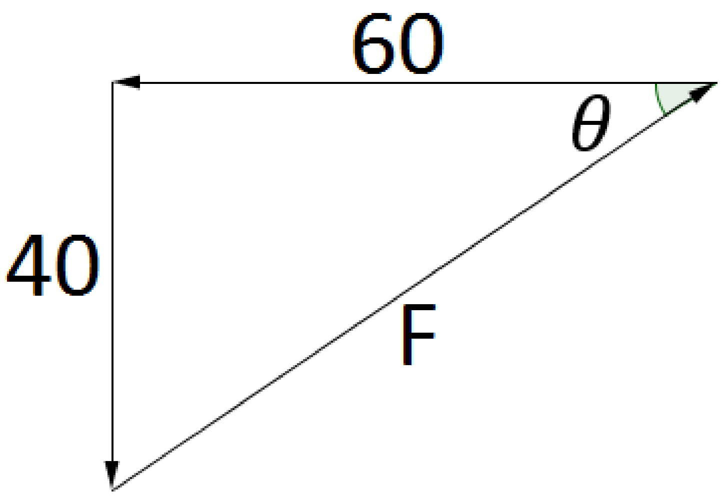
(a) Find F.

[2 marks]

(b) Find θ.

[3 marks]

2.
Alternative 1 (triangle of forces):



a)

$$F = \sqrt{60^2 + 40^2} = \mathbf{72.1\ N\ to\ 3\ s.f.}$$

b)

$$\tan\theta = \frac{40}{60} \quad \Rightarrow \quad \boldsymbol{\theta = 33.7° \ to\ 3\ s.f.}$$

Alternative 2 (resolving):
a) & b)
Resolving vertically:

$$F\sin\theta = 40$$

Resolving horizontally:

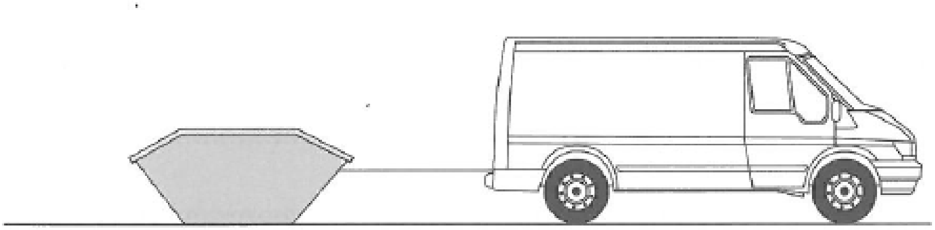
$$F\cos\theta = 60$$

Solving simultaneously:

$$\frac{F\sin\theta}{F\cos\theta} = \frac{40}{60} \quad \Rightarrow \quad \tan\theta = \frac{2}{3} \quad \Rightarrow \quad \boldsymbol{\theta = 33.7° \ to\ 3\ s.f.}$$

$$F = \frac{40}{\sin\theta} = \frac{40}{\sin 33.7} = \mathbf{72.1\ N\ to\ 3\ s.f.}$$

3 A skip, of mass 800 kg, is at rest on a rough horizontal surface. The coefficient of friction between the skip and the ground is 0.4. A rope is attached to the skip and then the rope is pulled by a van so that the rope is horizontal while it is taut, as shown in the diagram.



The mass of the van is 1700 kg. A constant horizontal forward driving force of magnitude P newtons acts on the van. The skip and the van accelerate at $0.05\ \text{m s}^{-2}$.

Model both the van and the skip as particles connected by a light inextensible rope. Assume that there is no air resistance acting on the skip or on the van.

(a) Find the speed of the van and the skip when they have moved 6 metres.

[3 marks]

(b) Draw a diagram to show the forces acting on the skip while it is accelerating.

[1 mark]

(c) Draw a diagram to show the forces acting on the van while it is accelerating. State one advantage of modelling the van as a particle when considering the vertical forces.

[2 marks]

(d) Find the magnitude of the friction force acting on the skip.

[3 marks]

(e) Find the tension in the rope.

[3 marks]

(f) Find P.

[3 marks]

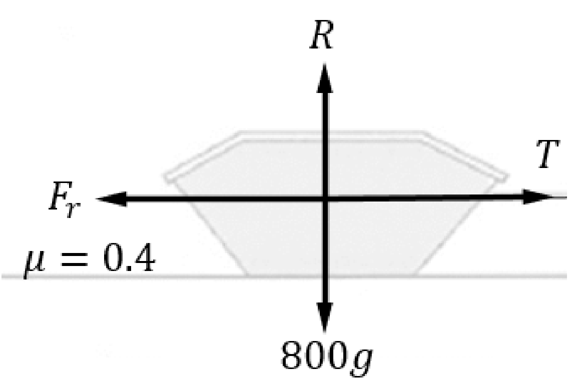
3.

a)

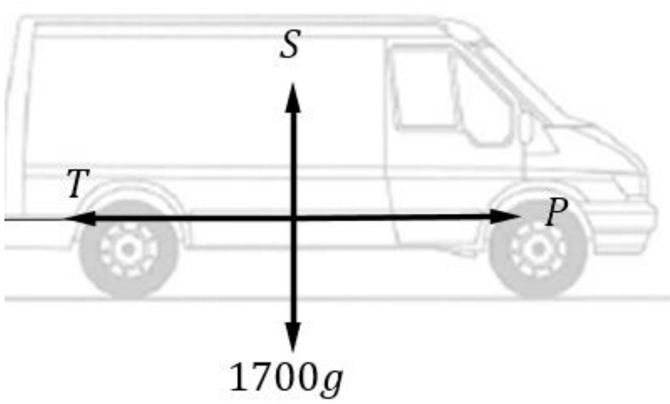
$$s = 6 \quad u = 0 \quad v = - \quad a = 0.05 \quad t = -$$

$$v^2 = u^2 + 2as = 0 + 2(0.05)(6) = 0.6 \quad \Rightarrow \quad v = \sqrt{0.6} = \mathbf{0.775\ ms^{-1}\ to\ 3\ s.f.}$$

b)



c)



An advantage of modelling the van as a particle when considering vertical forces is that the contact forces which in reality would act through the four wheels can be modelled as a single force, directly opposing the weight.

d)
Resolving vertically at the skip:

$$R = 800g$$

Using $F_r = \mu R$:

$$F_r = 0.4 \times 800g = \mathbf{3136\ N}$$

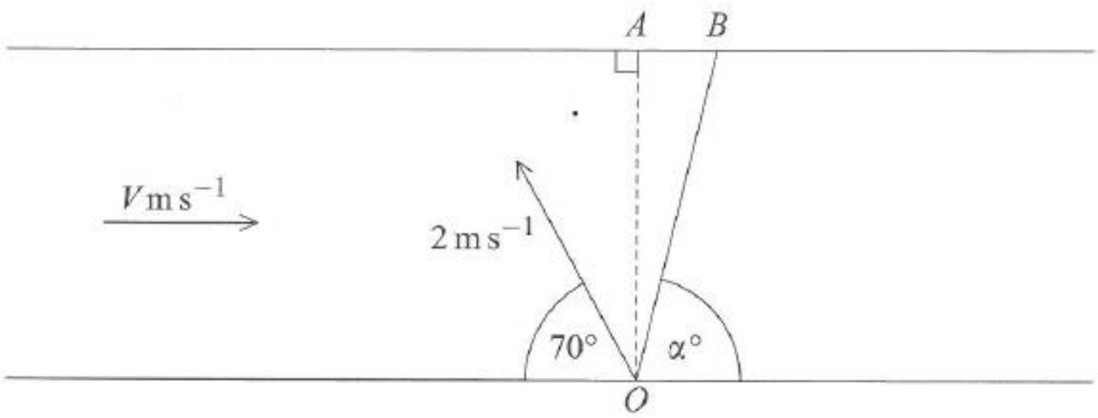
e)
Resolving horizontally at the skip:

$$F = ma \quad \Rightarrow \quad T - F_r = 800 \times 0.05 \quad \Rightarrow \quad T - 3136 = 40 \quad \Rightarrow \quad \boldsymbol{T = 3176\ N}$$

f)
Resolving horizontally at the van:

$$F = ma \quad \Rightarrow \quad P - T = 1700 \times 0.05 \quad \Rightarrow \quad P = 85 + 3176 = \mathbf{3261\ N}$$

4 A boat is crossing a river, which has two parallel banks. The width of the river is 20 metres. The water in the river is flowing at a speed of $V\,\mathrm{m\,s^{-1}}$. The boat sets off from the point O on one bank. The point A is directly opposite O on the other bank. The velocity of the boat relative to the water is $2\,\mathrm{m\,s^{-1}}$ at an angle of 70° to the bank. The boat lands at the point B which is 3 metres from A. The angle between the actual path of the boat and the bank is $\alpha°$. The river and the velocities are shown in the diagram.



(a) Find the time that it takes for the boat to cross the river.

[3 marks]

(b) Find α.

[2 marks]

(c) Find V.

[5 marks]

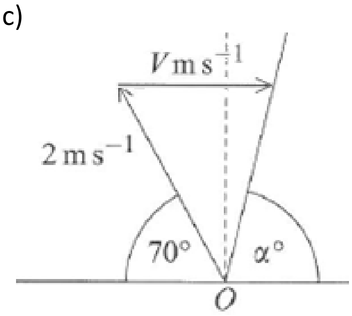
4.
a)
Resolving perpendicular to the bank: $v = 2\sin 70 \quad x = 20 \quad t = -$

$$v = \frac{x}{t} \quad \Rightarrow \quad 2\sin 70 = \frac{20}{t} \quad \Rightarrow \quad t = \frac{20}{2\sin 70} = \mathbf{10.6\ s\ to\ 3\ s.f.}$$

b)

$$A\hat{B}O = \alpha \quad \Rightarrow \quad \tan\alpha = \frac{20}{3} \quad \Rightarrow \quad \alpha = \tan^{-1}\frac{20}{3} = \mathbf{81.5°\ to\ 3\ s.f.}$$

c)



$$\frac{a}{\sin A} = \frac{b}{\sin B} \quad \Rightarrow \quad \frac{2}{\sin\alpha} = \frac{V}{\sin(110 - \alpha)}$$

$$\Rightarrow \quad V = \frac{2\sin 28.53\ldots}{\sin 81.46\ldots} = \mathbf{0.966\ ms^{-1}\ to\ 3\ s.f.}$$

5 Two particles, A and B, have masses of m and km respectively, where k is a constant. The particles are moving on a smooth horizontal plane when they collide and coalesce to form a single particle. Just before the collision the velocities of A and B are $(4\mathbf{i}+2\mathbf{j})\,\mathrm{m\,s^{-1}}$ and $(6\mathbf{i}-2\mathbf{j})\,\mathrm{m\,s^{-1}}$ respectively. Immediately after the collision the combined particle has velocity $(5.2\mathbf{i}-0.4\mathbf{j})\,\mathrm{m\,s^{-1}}$.

Find k.

[5 marks]

5.

Using conservation of momentum:

$$m\begin{bmatrix}4\\2\end{bmatrix}+km\begin{bmatrix}6\\-2\end{bmatrix}=(m+km)\begin{bmatrix}5.2\\-0.4\end{bmatrix}$$

$$\Rightarrow\quad \begin{bmatrix}4m+6km\\2m-2km\end{bmatrix}=\begin{bmatrix}5.2(1+k)m\\-0.4(1+k)m\end{bmatrix}\quad\Rightarrow\quad 4+6k=5.2+5.2k\quad and\quad 2-2k=-0.4-0.4k$$

$$\Rightarrow\quad 0.8k=1.2\quad\Rightarrow\quad \boldsymbol{k=1.5}\quad and\quad 2.4=1.6k\quad\Rightarrow\quad \boldsymbol{k=1.5}$$

6 A bullet is fired from a rifle at a target, which is at a distance of 420 metres from the rifle. The bullet leaves the rifle travelling at $V\,\mathrm{m\,s^{-1}}$ and at an angle of $2°$ above the horizontal. The centre of the target, C, is at the same horizontal level as the rifle. The bullet hits the target at the point A, which is on a vertical line through C. The bullet takes 1.8 seconds to reach the point A.

(a) Find V, showing clearly how you obtain your answer.

[3 marks]

(b) Find the distance between A and C.

[4 marks]

(c) State one assumption that you have made about the forces acting on the bullet.

[1 mark]

6.

a)

Horizontal motion:

$$v=V\cos 2\quad x=420\quad t=1.8$$

$$v=\frac{x}{t}\quad\Rightarrow\quad V\cos 2=\frac{420}{1.8}\quad\Rightarrow\quad V=\frac{420}{1.8\cos 2}=\boldsymbol{233\ ms^{-1}\ to\ 3\ s.f.}$$

b)

Vertical motion:

$$s=-\quad u=233\sin 2\quad v=-\quad a=-9.8\quad t=1.8$$

$$s=ut+\frac{1}{2}at^2\quad\Rightarrow\quad s=233\sin 2\times 1.8+\frac{1}{2}(-9.8)(1.8^2)=-1.209\ldots\quad\Rightarrow\quad Distance\ AC=\boldsymbol{1.21\ m\ to\ 3\ s.f.}$$

c)

The only force acting on the bullet is weight. For instance, no air resistance acts on the bullet.

7 Two particles, A and B, move on a horizontal surface with constant accelerations of $-0.4\mathbf{i}\,\mathrm{m\,s^{-2}}$ and $0.2\mathbf{j}\,\mathrm{m\,s^{-2}}$ respectively. At time $t=0$, particle A starts at the origin with velocity $(4\mathbf{i}+2\mathbf{j})\,\mathrm{m\,s^{-1}}$. At time $t=0$, particle B starts at the point with position vector $11.2\mathbf{i}$ metres, with velocity $(0.4\mathbf{i}+0.6\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) Find the position vector of A, 10 seconds after it leaves the origin.

[2 marks]

(b) Show that the two particles collide, and find the position vector of the point where they collide.

[9 marks]

7.

a)

Particle A:

$$s=-\quad u=\begin{bmatrix}4\\2\end{bmatrix}\quad v=-\quad a=\begin{bmatrix}-0.4\\0\end{bmatrix}\quad t=t\quad x_0=\begin{bmatrix}0\\0\end{bmatrix}$$

$$s=ut+\frac{1}{2}at^2+x_0\quad\Rightarrow\quad s=\begin{bmatrix}4\\2\end{bmatrix}t+\frac{1}{2}\begin{bmatrix}-0.4\\0\end{bmatrix}t^2+\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}4t-0.2t^2\\2t\end{bmatrix}$$

$$t=10\quad\Rightarrow\quad s=\begin{bmatrix}4(10)-0.2(10)^2\\2(10)\end{bmatrix}=\begin{bmatrix}\mathbf{20}\\\mathbf{20}\end{bmatrix}$$

Note: it is not necessary to find the general form for displacement at time t for part a), but it is needed for part b).

b)

Particle B:

$$s=-\quad u=\begin{bmatrix}0.4\\0.6\end{bmatrix}\quad v=-\quad a=\begin{bmatrix}0\\0.2\end{bmatrix}\quad t=t\quad x_0=\begin{bmatrix}11.2\\0\end{bmatrix}$$

$$s=ut+\frac{1}{2}at^2+x_0\quad\Rightarrow\quad s=\begin{bmatrix}0.4\\0.6\end{bmatrix}t+\frac{1}{2}\begin{bmatrix}0\\0.2\end{bmatrix}t^2+\begin{bmatrix}11.2\\0\end{bmatrix}=\begin{bmatrix}0.4t+11.2\\0.6t+0.1t^2\end{bmatrix}$$

If the particles collide, there is a value of t for which:

$$\begin{bmatrix}4t-0.2t^2\\2t\end{bmatrix}=\begin{bmatrix}0.4t+11.2\\0.6t+0.1t^2\end{bmatrix}$$

Equating the i components:

$$4t-0.2t^2=0.4t+11.2\quad\Rightarrow\quad t^2-18t+56=0\quad\Rightarrow\quad(t-14)(t-4)=0\quad\Rightarrow\quad t=4\quad or\quad t=14$$

Equating the j components:

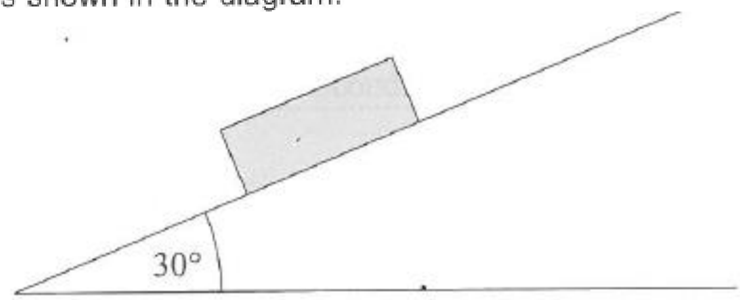
$$2t=0.6t+0.1t^2\quad\Rightarrow\quad t^2-14t=0\quad\Rightarrow\quad t(t-14)=0\quad\Rightarrow\quad t=0\quad or\quad t=14$$

A collision takes place only when i components and j components are equal. Therefore $t=14$.

Position of collision:

$$\begin{bmatrix}4t-0.2t^2\\2t\end{bmatrix}=\begin{bmatrix}4\times14-0.2(14)^2\\2\times14\end{bmatrix}=\begin{bmatrix}\mathbf{16.8}\\\mathbf{28}\end{bmatrix}$$

8 A crate, of mass 40 kg, is initially at rest on a rough slope inclined at $30°$ to the horizontal, as shown in the diagram.

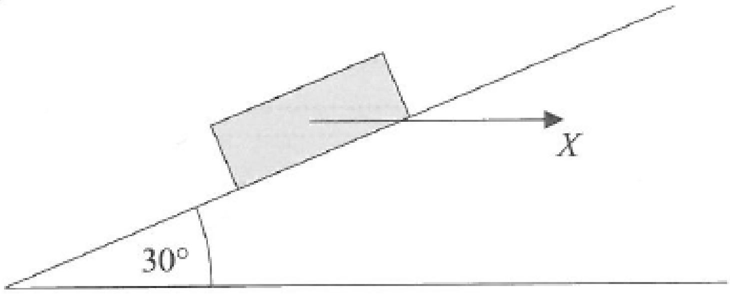


The coefficient of friction between the crate and the slope is μ.

(a) Given that the crate is on the point of slipping down the slope, find μ.

[5 marks]

(b) A horizontal force of magnitude X newtons is now applied to the crate, as shown in the diagram.



(i) Find the normal reaction on the crate in terms of X.

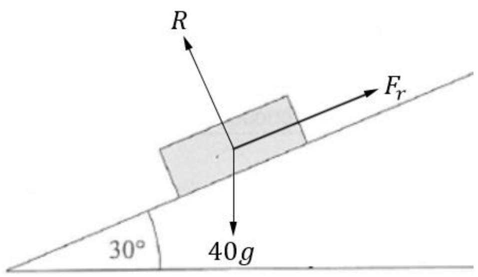
[2 marks]

(ii) Given that the crate accelerates up the slope at $0.2\,\mathrm{m\,s^{-2}}$, find X.

[5 marks]

8.

a)



Resolving perpendicular to the slope:

$$R = 40g\cos 30 = 339\ N\ to\ 3\ s.f.$$

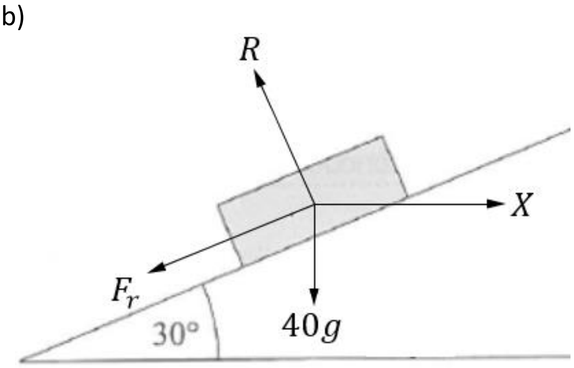
Resolving parallel to the slope:

$$F_r = 40g\sin 30 = 196\ N$$

Using $F_r = \mu R$ (in limiting equilibrium):

$$196 = 339.48\ldots\mu \quad \Rightarrow \quad \boldsymbol{\mu = 0.577\ to\ 3\ s.f.}$$

b)



i.

Resolving perpendicular to the slope:

$$R = 40g\cos 30 + X\sin 30 = \mathbf{339 + 0.5X}$$

ii.

Using $F_r = \mu R$: $F_r = 0.577(339 + 0.5X) = 196 + 0.289X$

Resolving up the slope:

$$X\cos 30 - 40g\sin 30 - F_r = 40 \times 0.2$$

$$\Rightarrow \quad X\cos 30 - 196 - (196 + 0.289X) = 8$$

$$\Rightarrow \quad X = \frac{400}{(\cos 30 - 0.289)} = \mathbf{693\ N\ to\ 3\ s.f.}$$