Volume and Surface Area Investigation

For this investigation you will need access to a variety of packages with a range of shapes and sizes.

You will need to find at least **three different objects** which you can measure and then calculate both volume and surface area. Any shape will do, but these are most common:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Volume</th>
<th>Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cuboid</td>
<td>(lwh)</td>
<td>(Area of all faces)</td>
</tr>
<tr>
<td>Prism</td>
<td>cross-sectional area (\times l)</td>
<td>(Area of all faces)</td>
</tr>
<tr>
<td>Pyramid</td>
<td>(\frac{1}{3} \times \text{base} \times h)</td>
<td>(Area of all faces)</td>
</tr>
<tr>
<td>Sphere</td>
<td>(\frac{4}{3} \pi r^3)</td>
<td>(4\pi r^2)</td>
</tr>
<tr>
<td>Cylinder</td>
<td>(\pi r^2 h)</td>
<td>(2\pi r^2 + 2\pi rh)</td>
</tr>
<tr>
<td>Cone</td>
<td>(\frac{1}{3} \pi r^2 h)</td>
<td>(\pi r^2 + \pi rl)</td>
</tr>
</tbody>
</table>

\(w = \text{width}\)  
\(l = \text{length}\)  
\(h = \text{height}\)  
\(r = \text{radius}\)  
\(l = \text{slant length}\)  
Note: \(r^2 + h^2 = l^2\)

For best results, pick an object, then a similarly shaped object of a different size, and a similarly sized object of a different shape. This will make it easier for you to draw conclusions from your results. However, it’s not essential, as you will have the chance to compare your results with others later on in class anyway.

For each object, follow these steps:

1. **Measure** all relevant lengths.

2. Draw a neat **diagram** with this information.

3. Calculate the **volume** of your shape (include units).
4. Calculate the **surface area** of your shape.

5. Work out the **surface area to volume ratio**.
   This is the size of the packaging **compared to** the capacity of the package.
   Use: \(\frac{\text{Surface Area}}{\text{Volume}}\)

Example: Cuboid

For a cuboid measuring 8 cm by 5.5 cm by 3 cm:

\[ V = 8 \times 5.5 \times 3 = 132 \text{ cm}^3 \]

Top & Bottom: \(2 \times (8 \times 5.5) = 88\) cm²
Front & Back: \(2 \times (8 \times 3) = 48\) cm²
Left & Right: \(2 \times (5.5 \times 3) = 33\) cm²

\[ \text{Surface Area} = 88 + 48 + 33 = 169 \text{ cm}^2 \]

\[ \frac{\text{Volume}}{\text{Surface Area}} = \frac{132}{169} \approx 1.28 \text{ cm}^2 \text{ per cm}^3 \]

A lower ratio means the packaging is more efficient: it takes less cardboard to hold just as much stuff.

When you have found the surface area to volume ratio for all of your shapes, you should make a note of anything interesting you find out. Any observations you make don’t have to be justified – they are just conjectures (things that may be true).
<table>
<thead>
<tr>
<th>Object 1:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Name the shape &amp; draw a diagram with all relevant measurements:</td>
<td></td>
</tr>
<tr>
<td>Find the volume (include units):</td>
<td></td>
</tr>
<tr>
<td>Find the surface area (with units):</td>
<td></td>
</tr>
<tr>
<td>Calculate the ratio $\frac{SA}{V}$:</td>
<td></td>
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<th>Object 2:</th>
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<tr>
<td>Name the shape &amp; draw a diagram with all relevant measurements:</td>
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<tr>
<td>Find the volume (include units):</td>
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</tr>
<tr>
<td>Find the surface area (with units):</td>
<td></td>
</tr>
<tr>
<td>Calculate the ratio $\frac{SA}{V}$:</td>
<td></td>
</tr>
</tbody>
</table>
Object 3:
Name the shape & draw a diagram
with all relevant measurements:

Find the volume (include units):

Find the surface area (with units):

Calculate the ratio $\frac{SA}{V}$:

**Observations / Conjectures / Questions**

Make a note of anything you noticed or that you want to understand better as a result of your investigations. Remember, a conjecture doesn’t have to be backed up by proof – it’s just an idea that may be true (it’s a tool for guiding further inquiry, nothing more).

Not sure how to begin? Try completing some of these sentences:

- It looks like...
- I wonder if...
- The larger the object...
- The cuboids I investigated...
- Compared to a similar sized cuboid, the cylinder...
- What would happen if I changed...?
Since this was an open investigation, specific solutions for object surface areas and volumes can obviously not be given, but some examples of objects are given below:

<table>
<thead>
<tr>
<th>Object</th>
<th>Shape</th>
<th>Volume</th>
<th>Surface Area</th>
<th>S.A. to Vol. ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>750g Cornflakes Packet</td>
<td>Cuboid</td>
<td>7717.5cm³</td>
<td>2786cm²</td>
<td>0.361 cm² per cm³</td>
</tr>
<tr>
<td>450g Ready Brek Packet</td>
<td>Cuboid</td>
<td>2092.5cm³</td>
<td>1153.5cm²</td>
<td>0.551 cm² per cm³</td>
</tr>
<tr>
<td>415g Heinz Baked Beans Tin</td>
<td>Cylinder</td>
<td>π(3.75²)(11) ≈ 486.0cm³</td>
<td>2π(3.75²) + 2π(3.75)(11) ≈ 347.5cm²</td>
<td>0.715 cm² per cm³</td>
</tr>
<tr>
<td>200g Apple</td>
<td>Sphere</td>
<td>4/3 π(3.5³) ≈ 179.6cm³</td>
<td>4π(3.5²) ≈ 153.9cm²</td>
<td>0.857 cm² per cm³</td>
</tr>
<tr>
<td>90ml Cornetto</td>
<td>Cone</td>
<td>π(3²)(12) ≈ 113.1cm³</td>
<td>π(3²) + π(3)(√3² + 12²) ≈ 144.9cm²</td>
<td>1.28 cm² per cm³</td>
</tr>
</tbody>
</table>

A few more:
- 9m³ (9000000cm³) shed: 0.0333 cm² per cm³
- 130cm³ staples box: 1.28 cm² per cm³
- 1cm³ marble: 4.84 cm² per cm³
- 1cm³ dice: 6 cm² per cm³
- Planet Earth: 0.00000000471 cm² per cm³

**A note on units**

Because volume increases more rapidly than area when you make things bigger, a cubic metre does not contain 100 cubic centimetres. In fact, 1m³ = 1000000cm³, and 1m² = 10000cm². For this reason, if surface area to volume ratio is measured in m² per m³, the values will be 100 times greater.
Volume and Surface Area Investigation  SOLUTIONS

While individual examples alone are not sufficient proof of the following results, they are beneficial in understanding and interpreting the underlying ideas.

**Key result 1: Size matters**
The best way to make your packaging more efficient is to make it bigger. In general, the larger the shape, the more efficient it is.

This is because when you enlarge a shape, volume increases more rapidly than surface area.

**Real application:**
Babies are about the same shape as adults, but because they are much smaller, their surface area to volume ratio is greater, so they lose body heat much faster if they’re not covered up.

*Roughly speaking, a new-born infant has a surface area to volume ratio approximately 3 times that of a fully grown adult.*

Comparing different sizes of cube, you can see volume increasing faster than surface area:

\[
\frac{SA}{Vol} = \frac{6}{1} = 6 \quad \frac{24}{8} = 3 \quad \frac{54}{27} = 2 \quad \frac{96}{64} = 1.5
\]

Note: Beyond a 6cm cube the ratio drops below 1 cm² per cm³. Eg for 10cm, 0.6 cm² per cm³.

Also, we used cubes here to make it easier to visualise, but remember this concept works for all 3D shapes, just because they are three-dimensional. When you ‘double’ the size of a 3D shape, you are doubling length, width and height, so you are doubling three times.

**Key result 2: Spheres rule**
For a fixed size, some shapes are more efficient than others. Cubes are better than cuboids, for instance. The very best shape is a sphere, so the more like a sphere a shape looks, the more efficient it will be.

**Real application:**
To survive, a bubble must contain a fixed volume of air without making its surface too thin. The optimal solution is a sphere.

*Spheres rule*
Similarly, since gravity is the dominant force on really large objects, it forces planets and stars to be as tightly packed as possible, making them spherical in shape.

*Earth is a more perfect sphere than a ping-pong ball.*

Since a square is the most efficient rectangle (perimeter to area ratio), when we extend from 2D to 3D, it makes sense that a cube should be the most efficient cuboid.

Similarly, since circles beat polygons, cones are better than any other pyramid.

For the same reason, cylinders are better than any other prism.

Overall, however, the sphere is the most efficient 3D shape.

If we directly compare these four common shapes for a fixed volume of 1 litre, we get:

<table>
<thead>
<tr>
<th>Cube*</th>
<th>Best Cone* ((h = 3r))</th>
<th>Best Cylinder* ((h = 2r))</th>
<th>Sphere*</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Vol = 1000cm^3)</td>
<td>(Vol = 1000cm^3)</td>
<td>(Vol = 1000cm^3)</td>
<td>(Vol = 1000cm^3)</td>
</tr>
<tr>
<td>(SA = 600cm^2)</td>
<td>(SA = 610cm^2)</td>
<td>(SA = 554cm^2)</td>
<td>(SA = 484cm^2)</td>
</tr>
<tr>
<td>*better than any cuboid</td>
<td>*better than any pyramid</td>
<td>*better than any prism</td>
<td>*better than any other shape</td>
</tr>
</tbody>
</table>