Volume and Surface Area Investigation

For this investigation you will need access to a variety of packages with a range of shapes and sizes.

You will need to find at least **three different objects** which you can measure and then calculate both volume and surface area. Any shape will do, but these are most common:

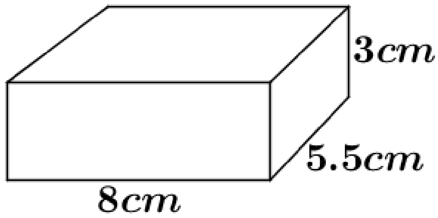
Shape	Volume	Surface Area	
Cuboid	lwh	(Area of all faces)	$w = width$
Prism	$cross\text{-}sectional\ area \times l$	(Area of all faces)	$l = length$
Pyramid	$\frac{1}{3} \times base \times h$	(Area of all faces)	$h = height$
Sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$	$r = radius$
Cylinder	$\pi r^2 h$	$2\pi r^2 + 2\pi rh$	$l = slant\ length$
Cone	$\frac{1}{3}\pi r^2 h$	$\pi r^2 + \pi rl$	Note: $r^2 + h^2 = l^2$

For best results, pick an object, then a similarly *shaped* object of a different *size*, and a similarly *sized* object of a different *shape*. This will make it easier for you to draw conclusions from your results. However, it's not essential, as you will have the chance to compare your results with others later on in class anyway.

For each object, follow these steps:

1. **Measure** all relevant lengths.
2. Draw a neat **diagram** with this information.
3. Calculate the **volume** of your shape (include units).
4. Calculate the **surface area** of your shape.
5. Work out the **surface area to volume ratio**.
This is the size of the packaging *compared to* the capacity of the package. Use: $\frac{Surface\ Area}{Volume}$

Example: Cuboid



$V = 8 \times 5.5 \times 3 = \mathbf{132cm^3}$

Top & Bottom: $2 \times (8 \times 5.5)$

Front & Back: $2 \times (8 \times 3)$

Left & Right: $2 \times (5.5 \times 3)$

$88 + 48 + 33 = \mathbf{169cm^2}$

$$\frac{Surface\ Area}{Volume} = \frac{169}{132}$$

$\approx \mathbf{1.28\ cm^2\ per\ cm^3}$

A *lower* ratio means the packaging is *more efficient:*
it takes *less* cardboard to hold *just as much* stuff.

When you have found the surface area to volume ratio for all of your shapes, you should make a note of anything interesting you find out. Any observations you make don't have to be justified – they are just *conjectures* (things that *may* be true).

Use the space below to record your investigation

(if you investigate more than 3 shapes, or just need extra space, feel free to use your book)

Object 1:
Name the shape & draw a diagram
with all relevant measurements:

Find the volume (include units):

Find the surface area (with units):

Calculate the ratio $\frac{SA}{V}$:

Object 2:
Name the shape & draw a diagram
with all relevant measurements:

Find the volume (include units):

Find the surface area (with units):

Calculate the ratio $\frac{SA}{V}$:

Object 3:
Name the shape & draw a diagram
with all relevant measurements:

Find the volume (include units):

Find the surface area (with units):

Calculate the ratio $\frac{SA}{V}$:

Observations / Conjectures / Questions

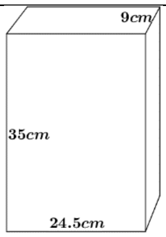
Make a note of anything you noticed or that you want to understand better as a result of your investigations. Remember, a conjecture *doesn't have to be backed up by proof – it's just an idea that* may *be true (it's a tool for guiding further inquiry, nothing more).*

Not sure how to begin? Try completing some of these sentences:

It looks like… I wonder if… The larger the object… The cuboids I investigated…
Compared to a similar sized cuboid, the cylinder… What would happen if I changed…?

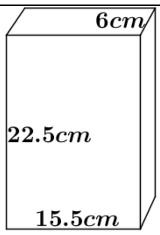
Volume and Surface Area Investigation SOLUTIONS

Since this was an open investigation, specific solutions for object surface areas and volumes can obviously not be given, but some examples of objects are given below:



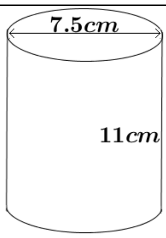
750g Cornflakes Packet: **Cuboid**
Vol. : $\mathbf{7717.5cm^3}$
S.A. : $\mathbf{2786cm^2}$

S.A. to Vol. ratio: $\mathbf{0.361\ cm^2\ per\ cm^3}$



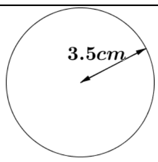
450g Ready Brek Packet: **Cuboid**
Vol. : $\mathbf{2092.5cm^3}$
S.A. : $\mathbf{1153.5cm^2}$

S.A. to Vol. ratio: $\mathbf{0.551\ cm^2\ per\ cm^3}$



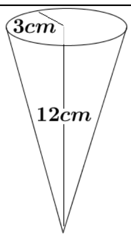
415g Heinz Baked Beans Tin: **Cylinder**
Vol. : $\pi(3.75^2)(11) \approx \mathbf{486.0cm^3}$
S.A. : $2\pi(3.75^2) + 2\pi(3.75)(11) \approx \mathbf{347.5cm^2}$

S.A. to Vol. ratio: $\mathbf{0.715\ cm^2\ per\ cm^3}$

200g Apple: **Sphere**
Vol. : $\frac{4}{3}\pi(3.5^3) \approx \mathbf{179.6cm^3}$
S.A. : $4\pi(3.5^2) \approx \mathbf{153.9cm^2}$

S.A. to Vol. ratio: $\mathbf{0.857\ cm^2\ per\ cm^3}$



$90ml$ Cornetto: **Cone**
Vol. : $\pi(3^2)(12) \approx \mathbf{113.1cm^3}$
S.A. : $\pi(3^2) + \pi(3)\left(\sqrt{3^2 + 12^2}\right) \approx \mathbf{144.9cm^2}$

S.A. to Vol. ratio: $\mathbf{1.28\ cm^2\ per\ cm^3}$

A few more:

$9m^3$ ($9000000cm^3$) shed: $\mathbf{0.0333\ cm^2\ per\ cm^3}$
$130cm^3$ staples box: $\mathbf{1.28\ cm^2\ per\ cm^3}$
$1cm^3$ marble: $\mathbf{4.84\ cm^2\ per\ cm^3}$
$1cm^3$ dice: $\mathbf{6\ cm^2\ per\ cm^3}$
Planet Earth: $\mathbf{0.00000000471\ cm^2\ per\ cm^3}$

<u>A note on units</u>

Because volume increases more rapidly than area when you make things bigger, a cubic metre does not contain 100 cubic centimetres. In fact, $1m^3 = 1000000cm^3$, and $1m^2 = 10000cm^2$. For this reason, if surface area to volume ratio is measured in m^2 per m^3, the values will be 100 times greater.

Volume and Surface Area Investigation SOLUTIONS

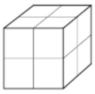
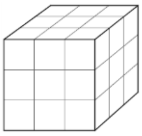
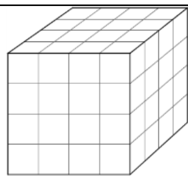
While individual examples alone are not sufficient proof of the following results, they are beneficial in understanding and interpreting the underlying ideas.

Key result 1: ***Size matters***

The best way to make your packaging more efficient is to make it bigger. In general, the larger the shape, the more efficient it is.
This is because when you enlarge a shape, volume increases more rapidly than surface area.

Real application:
Babies are about the same shape as adults, but because they are much *smaller*, their surface area to volume ratio is *greater*, so they lose body heat much faster if they're not covered up.
Roughly speaking, a new-born infant has a surface area to volume ratio approximately 3 times that of a fully grown adult.

Comparing different sizes of cube, you can see volume increasing faster than surface area:

			
$\frac{SA}{Vol} = \frac{6}{1} = 6$	$\frac{SA}{Vol} = \frac{24}{8} = 3$	$\frac{SA}{Vol} = \frac{54}{27} = 2$	$\frac{SA}{Vol} = \frac{96}{64} = 1.5$

Note: Beyond a $6cm$ cube the ratio drops below $1\ cm^2\ per\ cm^3$. Eg for $10cm$, $0.6\ cm^2\ per\ cm^3$
Also, we used cubes here to make it easier to visualise, but remember this concept works for all 3D shapes, just because they are three-dimensional. When you 'double' the size of a 3D shape, you are doubling length, width and height, so you are doubling three times.

Key result 2: ***Spheres rule***

For a fixed size, some shapes are more efficient than others. Cubes are better than cuboids, for instance.
The very best shape is a sphere, so the more like a sphere a shape looks, the more efficient it will be.

Real application:
To survive, a bubble must contain a fixed volume of air without making its surface too thin. The optimal solution is a sphere.
Similarly, since gravity is the dominant force on really large objects, it forces planets and stars to be as tightly packed as possible, making them spherical in shape.
Earth is a more perfect sphere than a ping-pong ball.

Since a square is the most efficient rectangle (perimeter to area ratio), when we extend from 2D to 3D, it makes sense that a **cube** should be the most efficient cuboid.
Similarly, since circles beat polygons, **cones** are better than any other pyramid.
For the same reason, **cylinders** are better than any other prism.
Overall, however, the **sphere** is the most efficient 3D shape.

If we directly compare these four common shapes for a fixed volume of $1\ litre$, we get:

Cube*	**Best Cone*** $(h = 3r)$	**Best Cylinder*** $(h = 2r)$	**Sphere***
$Vol = 1000cm^3$	$Vol = 1000cm^3$	$Vol = 1000cm^3$	$Vol = 1000cm^3$
$SA = \mathbf{600cm^2}$	$SA = \mathbf{610cm^2}$	$SA = \mathbf{554cm^2}$	$SA = \mathbf{484cm^2}$
**better than any cuboid*	**better than any pyramid*	**better than any prism*	**better than any other shape*