### Slicing Shapes

<table>
<thead>
<tr>
<th>Shape</th>
<th>Volume</th>
<th>Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>( \frac{4}{3} \pi r^3 )</td>
<td>( 4 \pi r^2 )</td>
</tr>
<tr>
<td>Cylinder</td>
<td>( \pi r^2 h )</td>
<td>( 2 \pi r^2 + 2 \pi rh )</td>
</tr>
</tbody>
</table>

Each end is a circle of radius \( r \), and the curved face is a rectangle of length \( h \) and width \( 2 \pi r \). 

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**A sphere of radius 3m is to be cut in half to increase the overall surface area.**

- Calculate the original surface area of the whole sphere:
- Calculate the additional surface area this circular cut provides:
- By what proportion has the surface area increased as a result of the cut?

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**A cylinder of radius 3m and height 4m is to be cut in half parallel to the ends.**

- Calculate the original surface area of the whole cylinder:
- Calculate the additional surface area this circular cut provides:
- By what proportion has the surface area increased as a result of the cut?

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**The cylinder (of radius 3m and height 4m) is instead cut in through the diameter.**

- Write down the original surface area of the whole cylinder:
- Calculate the additional surface area this rectangular cut provides:
- By what proportion has the surface area increased as a result of the cut?

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*By what proportion would the surface area be increased if you cut a cylinder in both these directions? What about cutting a sphere twice?*
**Slicing Shapes Extension**

**Volume:** \( \frac{1}{3} \pi r^2 h \)

**Surface Area:** \( \pi r^2 + \pi rl \)

(the base is a circle of radius \( r \), and the curved face is a sector of radius \( l \) and arc length \( 2\pi r \)).

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**A cone** of radius 3\( m \) and height 12\( m \) is to be cut vertically through the diameter.

Find the slant height and hence the original surface area of the whole cone:

Calculate the additional surface area this triangular cut provides.

By what proportion has the overall surface area increased as a result of the cut?

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**A cone** of radius 3\( m \) and height 12\( m \) is to be cut horizontally, parallel to the base, a distance of \( 12\sqrt{2} \)\( m \) from the top.

Write down the original surface area of the whole cone:

Calculate the additional surface area this circular cut provides:

Calculate the surface area of the frustum (truncated cone):

By what proportion has the overall surface area increased as a result of the cut?

*By what proportion would the surface area be increased if you cut a cone in both these directions?*
A sphere of radius $3\text{m}$ is to be cut in half to increase the overall surface area.

Calculate the original surface area of the whole sphere:

$$SA = 4\pi (3)^2 = 36\pi \text{m}^2$$

Calculate the additional surface area this circular cut provides:

Increased by two circles of radius $3$, so: $2\pi (3)^2 = 18\pi \text{m}^2$

By what proportion has the surface area increased as a result of the cut?

$$\frac{54\pi}{36\pi} = \frac{3}{2} \text{ or } 50\% \text{ increase}$$

A cylinder of radius $3\text{m}$ and height $4\text{m}$ is to be cut in half parallel to the ends.

Calculate the original surface area of the whole cylinder:

$$SA = 2\pi (3)^2 + 2\pi (3)(4) = 42\pi \text{m}^2$$

Calculate the additional surface area this circular cut provides:

Increased by two circles of radius $3$, so: $2\pi (3)^2 = 18\pi \text{m}^2$

By what proportion has the surface area increased as a result of the cut?

$$\frac{60\pi}{42\pi} = \frac{10}{7} \text{ or } \approx 43\% \text{ increase}$$

The cylinder (of radius $3\text{m}$ and height $4\text{m}$) is instead cut in through the diameter.

Write down the original surface area of the whole cylinder:

$$SA = 2\pi (3)^2 + 2\pi (3)(4) = 42\pi \text{m}^2$$

Calculate the additional surface area this rectangular cut provides:

Increased by two rectangles of width $6$ and length $4$, so:

$$2 \times 6 \times 4 = 48 \text{ m}^2$$

By what proportion has the surface area increased as a result of the cut?

$$\frac{42\pi + 48}{42\pi} = \frac{7\pi + 8}{7\pi} \text{ or } \approx 36\% \text{ increase}$$

Note: if you cut the cylinder in both directions as described, the S.A. increases by S.F. $\frac{10\pi + 8}{7\pi}$ or $\approx 79\% \text{ increase}$

If you cut a sphere in half twice, because you are adding four circles, you double the surface area (100% increase)
A cone of radius 3\(m\) and height 12\(m\) is to be cut vertically through the diameter.

Find the slant height and hence the original surface area of the whole cone:
\[ l = \sqrt{3^2 + 12^2} = 3\sqrt{17} \]
so:
\[ SA = \pi (3)^2 + \pi (3)(3\sqrt{17}) = 9\pi (1 + \sqrt{17}) \ m^2 \]

Calculate the additional surface area this triangular cut provides.
Base is 2\(\times\)3 and height is 12. Two triangles give an area of 72\(m^2\)

By what proportion has the overall surface area increased as a result of the cut?
\[ \frac{9\pi (1 + \sqrt{17}) + 72}{9\pi (1 + \sqrt{17})} \quad \text{or} \quad \approx 50\% \text{ increase} \]

A cone of radius 3\(m\) and height 12\(m\) is to be cut horizontally, parallel to the base, so as to leave a similar cone of half the volume as the top piece.

Write down the original surface area of the whole cone:
\[ 9\pi (1 + \sqrt{17}) \ m^2 \]

Calculate the additional surface area this circular cut provides:
Given a volume scale factor of 2 from the smaller cone to the larger, the length scale factor must be 2\(^{\frac{1}{3}}\) (or the cube root of 2), so the radius is \(3\sqrt{2}\), and the two circles we are adding have combined area:
\[ 2 \times \pi \left( 3 \times 2^{\frac{1}{3}} \right)^2 = 9\pi \times 2^3 \]

By what proportion has the overall surface area increased as a result of the cut?
\[ \frac{9\pi (1 + \sqrt{17}) + 9\pi \times 2^3}{9\pi (1 + \sqrt{17})} \quad \text{or} \quad \approx 62\% \text{ increase} \]

Note: if you cut the cone in half in both directions as described, the surface area increases by scale factor:
\[ \frac{9\pi (1 + \sqrt{17}) + 9\pi \times 2^3 + 72}{9\pi (1 + \sqrt{17})} \quad \text{or} \quad \approx 112\% \text{ increase (more than double)} \]

Direct comparison

<table>
<thead>
<tr>
<th>Shape</th>
<th>(r = 3m)</th>
<th>(r = 3m) (h = 4m)</th>
<th>(r = 3m) (h = 12m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>(36\pi \approx 113m^3)</td>
<td>(36\pi \approx 113m^3)</td>
<td>(36\pi \approx 113m^3)</td>
</tr>
<tr>
<td>Surface Area</td>
<td>(36\pi \approx 113m^2)</td>
<td>(42\pi \approx 132m^2)</td>
<td>(9\pi (1 + \sqrt{17}) \approx 145m^2)</td>
</tr>
<tr>
<td>% increase in S.A after two cuts</td>
<td>100%</td>
<td>79%</td>
<td>112%</td>
</tr>
<tr>
<td>New S.A.</td>
<td>(72\pi \approx 226m^2)</td>
<td>(60\pi + 48 \approx 236m^2)</td>
<td>(9\pi (1 + \sqrt{17}) + 9\pi \times 2^3 + 72 \approx 307m^2)</td>
</tr>
</tbody>
</table>