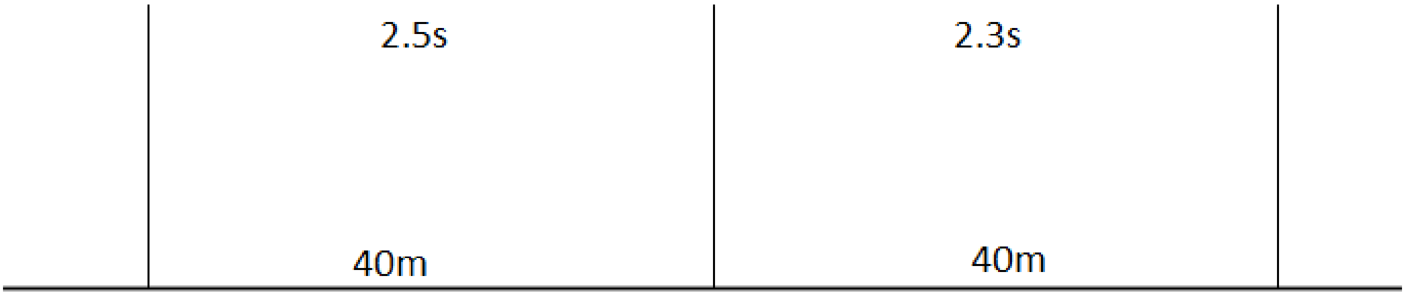
A tricky SUVAT problem

Telegraph poles, 40 m apart stand alongside a straight railway line. The times taken for a locomotive to pass the two gaps between three consecutive poles are 2.5 seconds and 2.3 seconds respectively. Assume that the acceleration of the train is constant. Calculate the acceleration of the train and the speed past the first post. *AQA M1 Textbook*

First, identify the key information and summarise it in a convenient form (eg a diagram):



Note the assumptions you have been given, or consider which you will need to make:

Constant acceleration means we can use $SUVAT$ equations, and that a velocity-time graph will resemble a straight line with gradient equal to the acceleration for the whole motion.

Make sure you know what you are trying to find:

We need the initial speed, and the acceleration of the train. Note that the train covers the second 40m stretch in a shorter time than the first, so it must have a positive acceleration.

List the known and needed variables, specifying the phase of motion being considered:

Defining the initial speed and acceleration (which the question asks for) as U and A:

Whole journey:	Between 1st two poles:	Between 2nd two poles:
$s = 80$	$s = 40$	$s = 40$
$u = U$	$u = U$	$u =$
$v =$	$v =$	$v =$
$a = A$	$a = A$	$a = A$
$t = 4.8$	$t = 2.5$	$t = 2.3$

Note: be wary of introducing too many variables too soon. Only label them as needed.

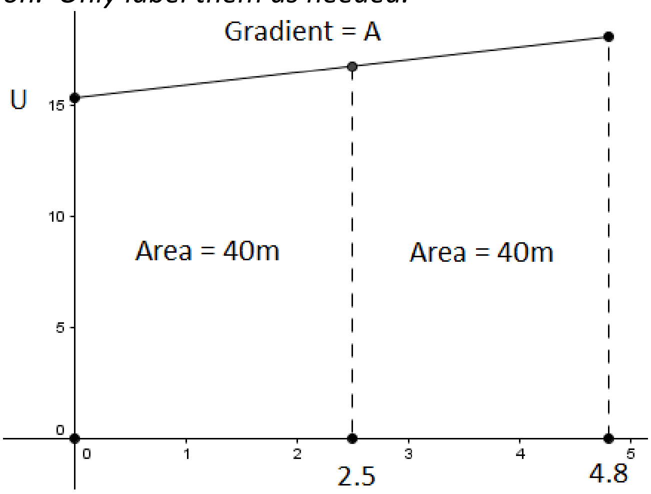
If the equation approach is confusing, sketch a quick velocity-time graph.

The SUVAT equations are built on the same basic concepts as:

$$Gradient = Acceleration$$

and:

$$Area = Displacement$$



Recall that SUVAT equations can link any four of the five variables together.

We don't have enough information from any one phase to directly calculate any of the unknowns, but by substituting the four variables of interest into the relevant formula we can form two simultaneous equations.

For the whole journey:

$$s = ut + \frac{1}{2}at^2 \quad \Longrightarrow \quad 80 = 4.8U + \frac{1}{2}A(4.8^2) \quad \Longrightarrow \quad 80 = 4.8U + 11.52A$$

Between the 1st two poles:

$$s = ut + \frac{1}{2}at^2 \quad \Longrightarrow \quad 40 = 2.5U + \frac{1}{2}A(2.5^2) \quad \Longrightarrow \quad 40 = 2.5U + 3.125A$$

Solve equations simultaneously, either using elimination if there is a convenient way to rearrange, or using direct (if messy) substitution:

Rearranging the first equation in terms of U:

$$U = \frac{80 - 11.52A}{4.8}$$

Substituting into the second equation:

$$40 = 2.5\left(\frac{80 - 11.52A}{4.8}\right) + 3.125A$$

Rearranging, simplifying and solving:

$$40 = 2.5\left(\frac{80 - 11.52A}{4.8}\right) + 3.125A$$

$$40 = \frac{125}{3} - 6A + 3.125A$$

$$2.875A = \frac{5}{3}$$

$$A = \frac{40}{69} = \boldsymbol{0.580ms^{-2}\ to\ 3\ s.f.}$$

Substituting back in to find U:

$$U = \frac{80 - 11.52\left(\frac{40}{69}\right)}{4.8} = \frac{1054}{69} = \boldsymbol{15.3ms^{-1}\ to\ 3\ s.f.}$$

Interpret your answers, or check that they are sensible & consistent with the situation:

For comparison, a fast car (0 to 60mph in 3 seconds) has an acceleration of $10ms^{-2}$. A train will have a considerably lower acceleration since it is so large. An acceleration of around $0.5ms^{-2}$ corresponds to 0 to 60mph (around 0 to 30 m/s) in around a minute.

A speed of $15.3ms^{-1}$ is roughly $30mph$, which seems reasonable for the speed of a train which is still accelerating. In addition, taking 4.8 seconds to travel 80 metres corresponds to an average speed of $16.\dot{6}ms^{-1}$, so a starting speed of $15.3ms^{-1}$ is consistent.