Hiker

A hiker sets off at 10am and walks at a steady speed for 2 hours due north, then turns and walks for a further 5 hours due west.

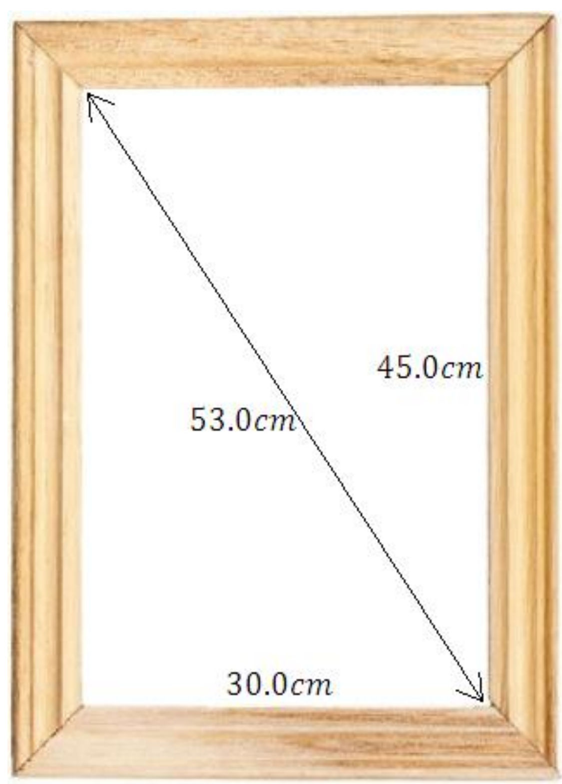
If he continues at the same speed, what's the earliest time he could arrive back at his starting point?

Framed

A rectangular picture frame is being constructed. To check that the shape is correct, measurements for the height, width and diagonal distance are measured.

The measurements are given opposite. Is the picture frame likely to be close enough to rectangular?

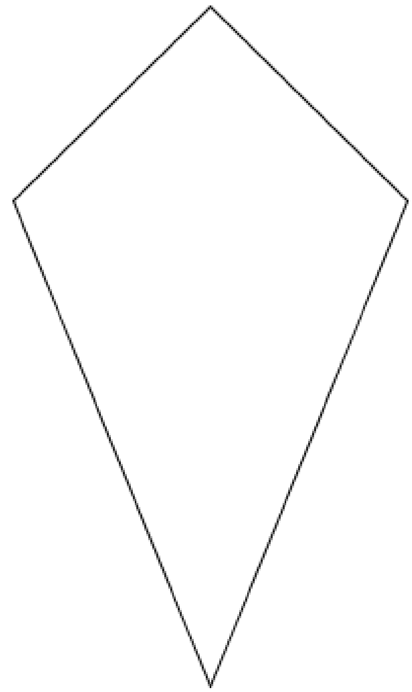
Give a reason for your answer.



Kite Area

Find the area of a kite of width $40cm$, longest side $52cm$ and shortest side $25cm$.

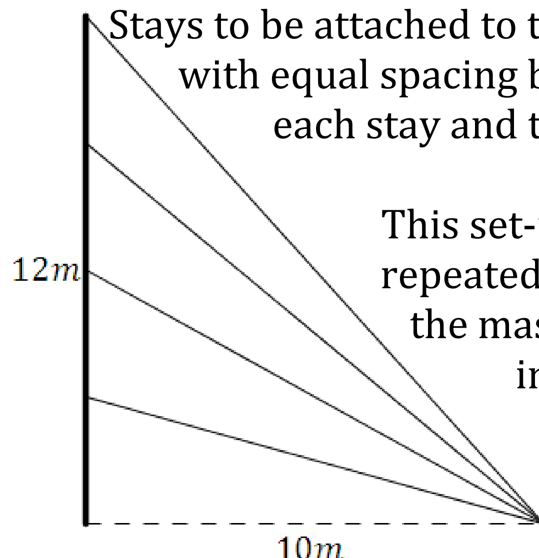
Hint: Divide into right-angled triangles.



Radio Mast

Amateur radio enthusiasts set up masts to transmit their signals. Because the mast itself is so thin, steel wires are used as stays to anchor the mast to the ground. A typical set-up is shown opposite.

Using the information given, calculate the total length of steel wire required to anchor the mast.



Stays to be attached to the mast with equal spacing between each stay and the next.

This set-up to be repeated around the mast at 90° intervals.

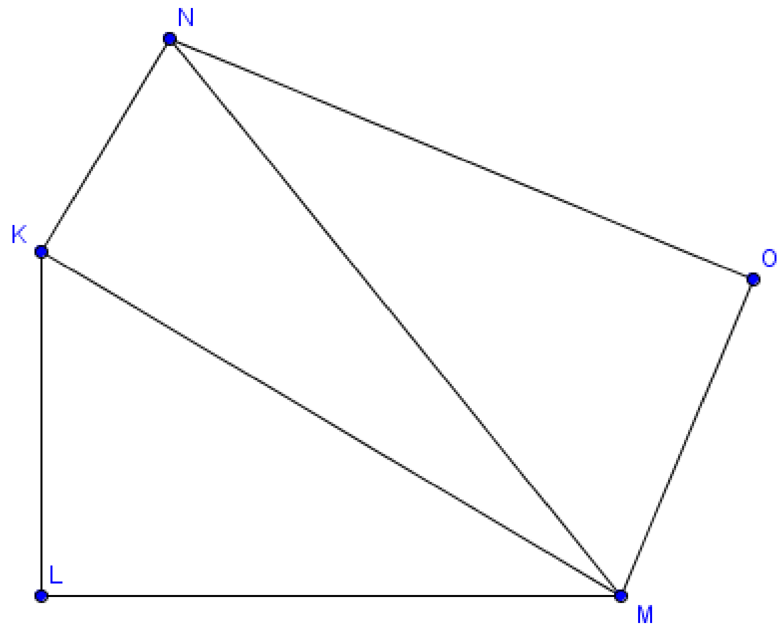
Not drawn to scale

Compound Triangles

In the diagram opposite, angles $K\hat{L}M$, $N\hat{K}M$ and $N\hat{O}M$ are all $90°$.

Given that $ON = 24cm$, $NK = 15cm$, $KL = 12cm$ and $LM = 16cm$, find the total outer perimeter of the shape.

Ext: Find the total area.



Not drawn to scale

Pringles Tube

Spaghetti is to be stored in the cylindrical Pringles tube shown opposite.

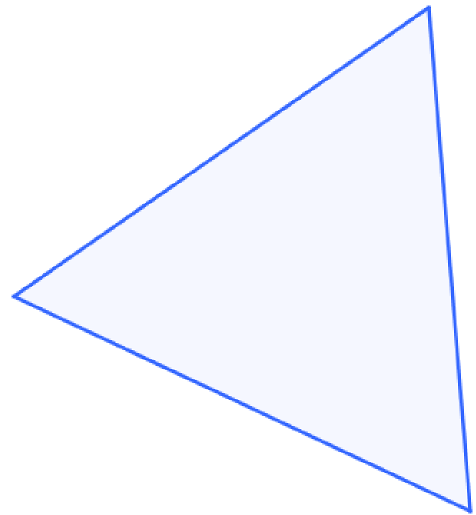
What is the maximum possible length for a piece of spaghetti if it is to fit in the tube?

Equilateral Area

Find the area of an equilateral triangle of side length $1m$.

Give your answer both as an exact value and as a decimal, rounded to 3 significant figures.

Hint: Cut the triangle in half and calculate the height.



TV Screen

The size of a flat-screen TV is usually given as the **length of the diagonal**.

A 42-inch flat-screen has aspect ratio 9:16 which means that the sides are of length $16x$ inches and $9x$ inches. Draw a triangle using this information and hence calculate the area of the screen.

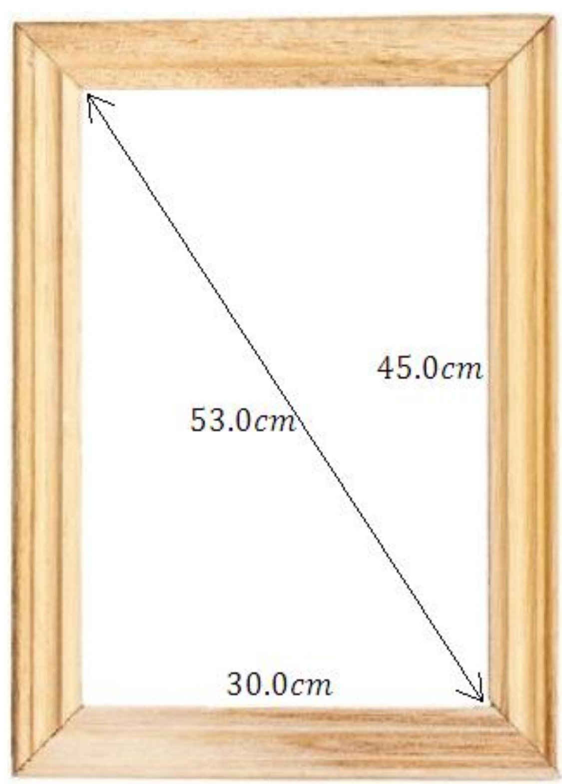
Ext:
How much bigger would the area be if the diagonal were 10% larger (44.2 in)?

Hiker HINTS

Because he is traveling at the same speed, you can use the number of hours as the lengths of the lines. Draw a diagram to show where he has walked already, then draw a straight line from his starting point to the end point. Work out the length of this line to calculate how many hours it would take him to walk. Finally, work out the time of day he would return.

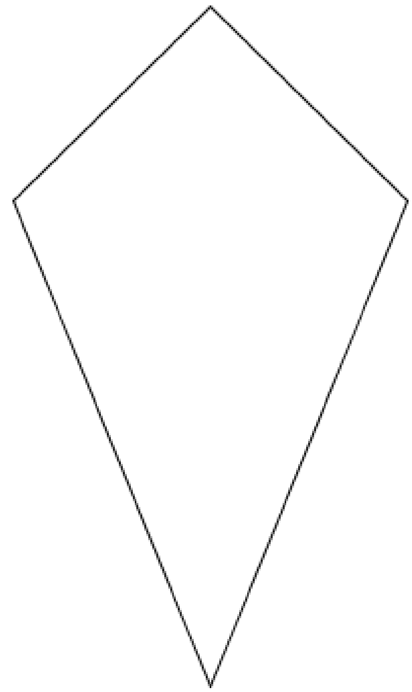
Framed HINTS

If the frame is a perfect rectangle, the corners will be at 90°. Use Pythagoras to calculate the length that the diagonal should be, then see if the distance given in the diagram is 'close enough' to the number you have calculated.



Kite Area HINTS

Split up the shape into right-angled triangles by joining up opposite corners. Mark on any lengths given in the question and use Pythagoras to work out missing lengths, then the area of each of the four triangles.

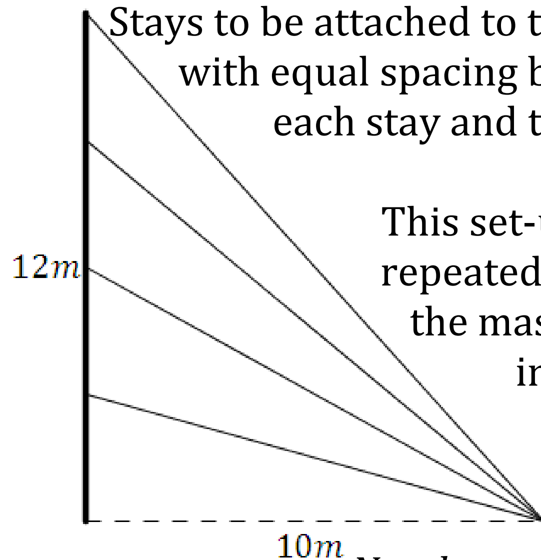


Radio Mast HINTS

This question involves four different right-angled triangles. Add as much information as possible to the diagram, then use Pythagoras to work out the length of each of the four wires. Remember the picture only shows one side of the mast: the same thing will be needed on four sides of the mast.

Stays to be attached to the mast with equal spacing between each stay and the next.

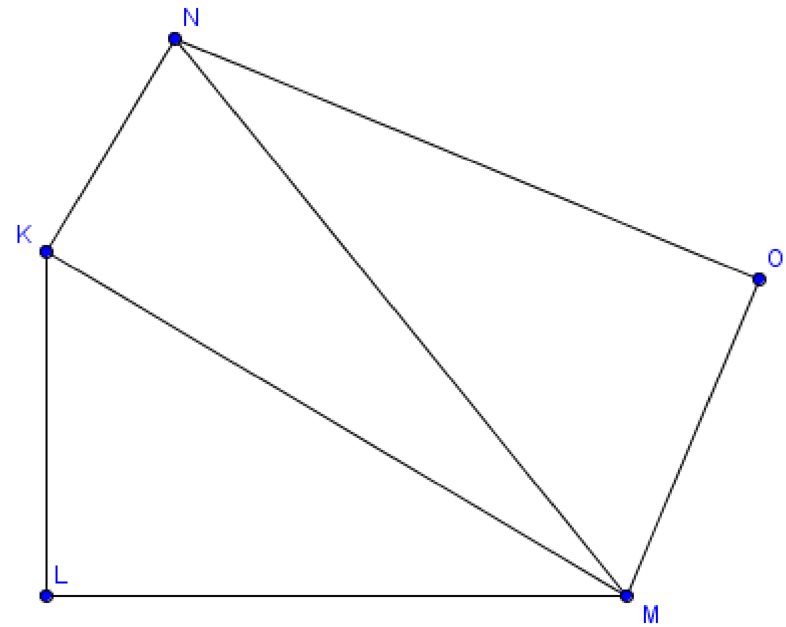
This set-up to be repeated around the mast at 90° intervals.



Not drawn to scale

Compound Triangles HINTS

Mark all the lengths you know on the diagram. You can use Pythagoras to find any unknown side for triangles where you know two of the sides already. Keep going until you know every side length. Add up the outer sides for the perimeter, or use the rule $A = \frac{1}{2} \times base \times height$ for each triangle to get area.



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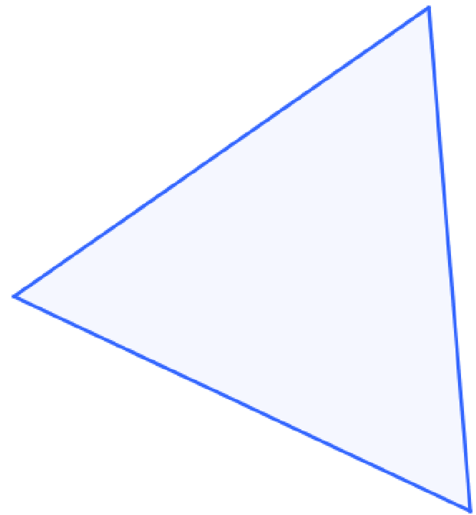
Pringles Tube HINTS

The longest straight line that can fit in the cylinder is a diagonal from the bottom left to top right (or vice versa). Make a right-angled triangle and use Pythagoras to work out the length of the hypotenuse.

Equilateral Area HINTS

If you cut an equilateral triangle in half, from one corner down to the midpoint of the opposite side, you get two identical right-angled triangles. Label all the lengths you know, and this should enable you to calculate the height. Then use $Area = \frac{1}{2}(base \times height)$ to calculate the area of the triangle.



TV Screen HINTS

Draw the triangle described, using the algebraic expressions for lengths as needed. Substitute these numbers into Pythagoras' theorem as usual (but in this case there will be x terms involved). Simplify and solve to find x. Remember: $(5x)^2$ is not the same as $5x^2$:
$(5x)^2 = 5x \times 5x = 5 \times 5 \times x \times x = 25x^2$

For the extension, to save you doing the same thing, use the fact that if the length increases by scale factor L, the area increases by scale factor L^2.

Hiker SOLUTIONS

The shortest route back is a straight line (hypotenuse of a right-angled triangle):

$$2^2 + 5^2 = x^2 \quad \Longrightarrow \quad x = \sqrt{29} \approx 5.385$$

$$5.385\ hrs = 5\ hrs\ 23.11\ mins$$

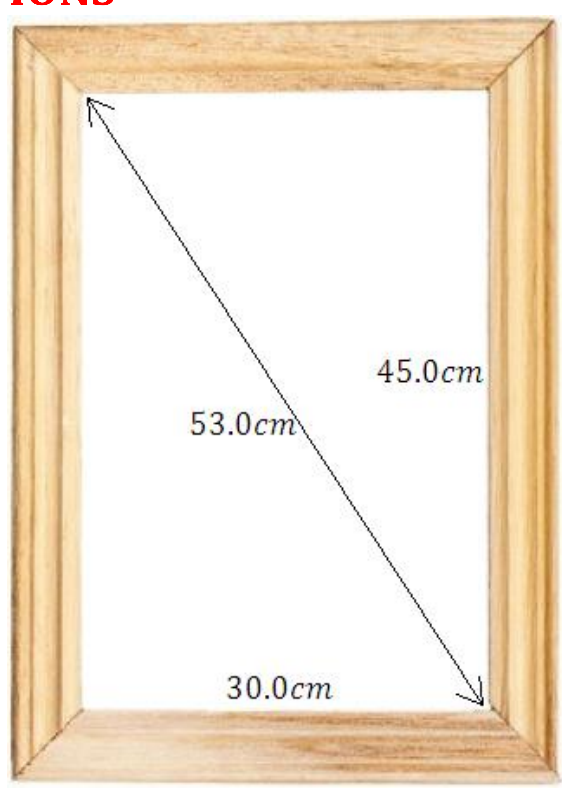
Total journey time: $12\ hrs\ 23.11\ mins$

Earliest return time: $\mathbf{22{:}23} \quad (\mathbf{10{:}23}\boldsymbol{pm})$

Framed SOLUTIONS

Diagonal:

$$30^2 + 45^2 = x^2 \quad \Longrightarrow \quad x = 54.083cm$$



The frame is not likely to be close enough to a true rectangle, since this value is not very close to the value it should be. If the measurements are taken – as they appear to be – to the nearest millimetre, the smallest the diagonal could be is:

$$\sqrt{29.95^2 + 44.95^2} = 54.014cm$$

(It can be shown that the angle in this triangle would be 92.5°.)

Kite Area SOLUTIONS

Top-right triangle:

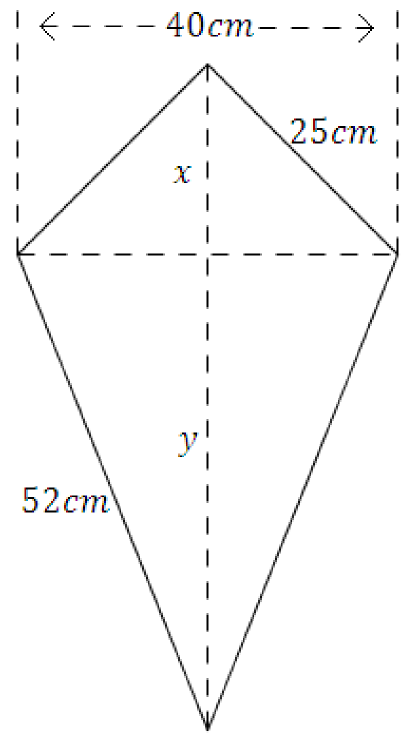
$$20^2 + x^2 = 25^2 \quad \Longrightarrow \quad x = 15$$

Bottom-left triangle:

$$20^2 + y^2 = 52^2 \quad \Longrightarrow \quad y = 48$$

Area:

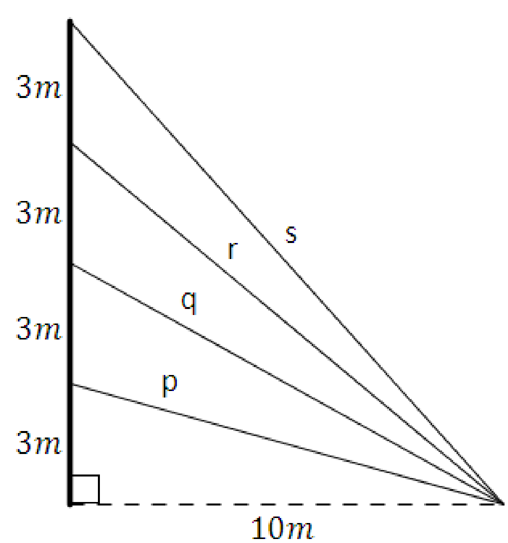
$$\frac{40 \times 15}{2} + \frac{40 \times 48}{2} = \mathbf{1260cm^2}$$



Radio Mast SOLUTIONS

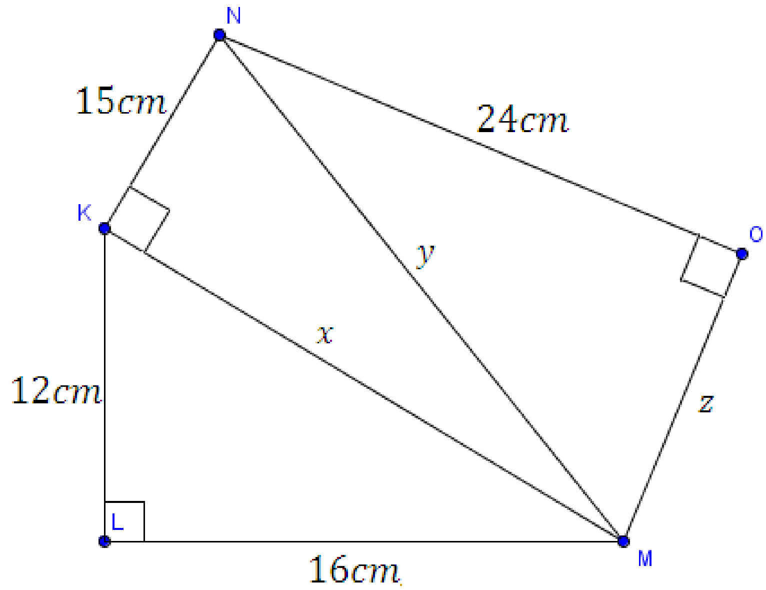
$$10^2 + 3^2 = p^2 \quad \Longrightarrow \quad p = \sqrt{109}$$
$$10^2 + 6^2 = q^2 \quad \Longrightarrow \quad q = \sqrt{136}$$
$$10^2 + 9^2 = r^2 \quad \Longrightarrow \quad r = \sqrt{181}$$
$$10^2 + 12^2 = s^2 \quad \Longrightarrow \quad s = \sqrt{244}$$

$$Total = 4(p + q + r + s) = \mathbf{204.7m}$$



Compound Triangles SOLUTIONS

$$12^2 + 16^2 = x^2 \quad \Longrightarrow \quad x = 20$$
$$15^2 + 20^2 = y^2 \quad \Longrightarrow \quad y = 25$$
$$24^2 + z^2 = 25^2 \quad \Longrightarrow \quad z = 7$$



Not drawn to scale

Perimeter:

$$24 + 15 + 12 + 16 + 7 = \mathbf{74cm}$$

Area:

$$\frac{12 \times 16}{2} + \frac{15 \times 20}{2} + \frac{24 \times 7}{2}$$

$$= \mathbf{330cm^2}$$

Pringles Tube SOLUTIONS

$$8^2 + 23^2 = x^2$$

$$\Longrightarrow \quad x = \mathbf{24.4cm} \text{ to } 1\ d.p.$$

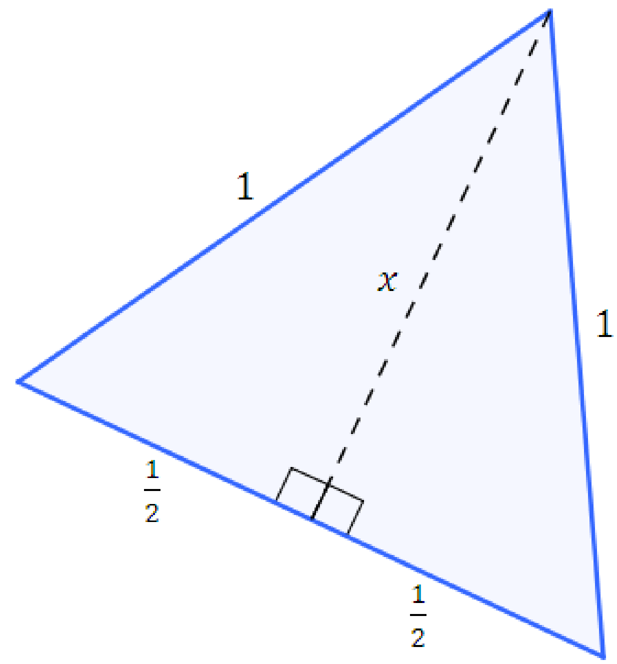
Equilateral Area SOLUTIONS

$$\left(\frac{1}{2}\right)^2 + x^2 = 1^2$$

$$\frac{1}{4} + x^2 = 1$$

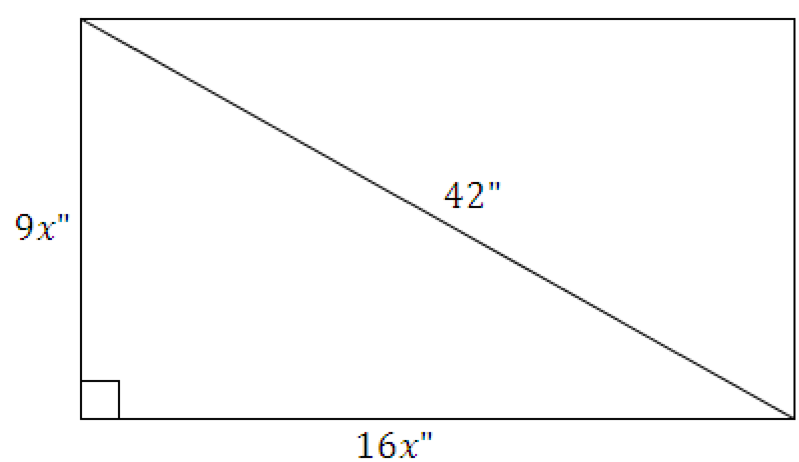
$$x^2 = \frac{3}{4} \quad \Longrightarrow \quad x = \sqrt{\left(\frac{3}{4}\right)} = \frac{\sqrt{3}}{2}$$

$$Area = \frac{1 \times \frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{\mathbf{3}}}{\mathbf{4}} \approx \mathbf{0.433m^2}$$



TV Screen SOLUTIONS

$$(9x)^2 + (16x)^2 = 42^2$$
$$81x^2 + 256x^2 = 1764$$
$$337x^2 = 1764$$
$$x^2 = \frac{1764}{337}$$
$$Area = (9x)(16x) = 144x^2$$
$$= 144\left(\frac{1764}{337}\right) = 753.8\ sq\ in.$$



Ext:

$$diagonal\ 1.1 \times longer \quad \Longrightarrow \quad area = 1.1^2 \times greater \quad \Longrightarrow \quad 912.0\ sq\ in.$$