Tracing an algorithm

When a computer follows a computer program, it carries out one instruction at a time (albeit very rapidly). To understand how a program functions it can be helpful to manually trace the algorithm; that is, to write down the outcome of every instruction as it comes along, just as a computer would.

Line 10: \( A = 1, \ B = \frac{1}{\sqrt{2}}, \ C = \frac{1}{4}, \ X = 1 \)
Line 20: Let \( Y = A \)
Line 30: Let \( A = \frac{A+B}{2} \)
Line 40: Let \( B = \sqrt{BY} \)
Line 50: Let \( C = C - X(A-Y)^2 \)
Line 60: Let \( X = 2X \)
Line 70: Print \( \frac{(A+B)^2}{4C} \)
Line 80: If \( X < 8 \) GoTo Line 20
Line 90: End

A trace of this algorithm involves making a note of each update to any variables used by the program (and often the printed outputs as well). Complete the trace:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>X</th>
<th>Y</th>
<th>Print</th>
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</thead>
<tbody>
<tr>
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<tr>
<td>0.853553</td>
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Can you see the aim? Designed by Yoshiaki Tamura and Yasumasa Kanada, in fewer than 20 passes it gives a result accurate to over 1 million decimal places!
Tracing an algorithm **SOLUTIONS**

When a computer follows a computer program, it carries out one instruction at a time (albeit very rapidly). To understand how a program functions it can be helpful to manually *trace* the algorithm; that is, to write down the outcome of every instruction as it comes along, just as a computer would.

Line 10: $A = 1, \ B = \frac{1}{\sqrt{2}}, \ C = \frac{1}{4}, \ X = 1$

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Line 30: Let $A = \frac{A + B}{2}$

Line 40: Let $B = \sqrt{BY}$

Line 50: Let $C = C - X(A - Y)^2$

Line 60: Let $X = 2X$

Line 70: Print $\frac{(A + B)^2}{4C}$

Line 80: If $X < 8$ GoTo Line 20

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