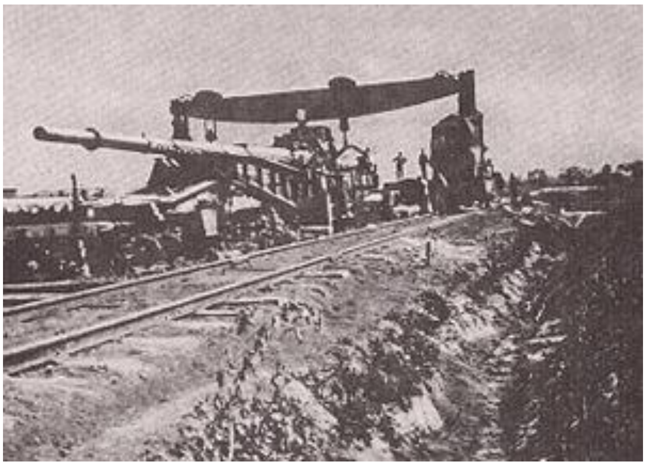
Paris Gun

The German-built Paris gun weighed $256\ tonnes$, was $34\ metres$ long and fired shells of mass $106\ kg$ at the phenomenal speed of $1640\ ms^{-1}$.



Its horizontal range was so great that it was used to fire on Paris from Germany, and its shells were the first man-made object to reach the stratosphere.

The usual angle of elevation for the gun was $55°$.

1. Calculate the total flight time, assuming the gun is fired over horizontal ground.

2. Find the horizontal range.

3. Calculate the maximum height reached by the shells.

4. Compare your results to the observational data below, and comment on any discrepancy. In particular, discuss any assumptions that you made and how your model may be refined.

Paris gun statistics :

Flight Time	**Range**	**Maximum Altitude**
$182\ seconds$	$130\ km$	$42.3\ km$

Paris Gun SOLUTIONS

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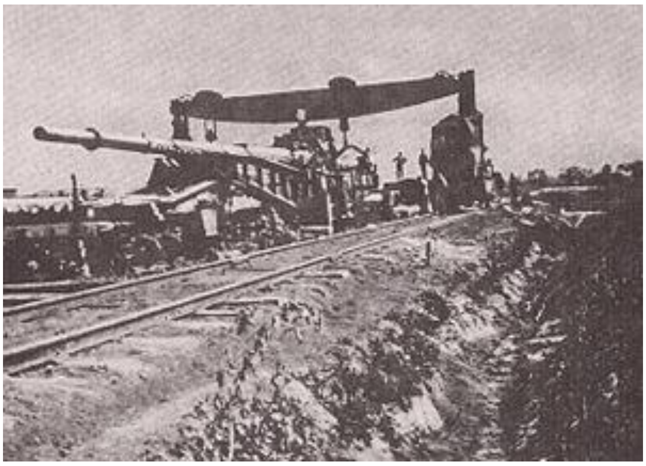

The usual angle of elevation for the gun was $55°$.

1. Calculate the total flight time, assuming the gun is fired over horizontal ground.

Vertical motion:

$$s = ut + \frac{1}{2}at^2 \quad \Longrightarrow \quad 0 = (1600\sin 55)t + \frac{1}{2}(-9.8)t^2 \quad \Longrightarrow \quad t = 0\ \ or\ \ \boldsymbol{t = 267s\ to\ 3\ s.f.}$$

2. Find the horizontal range.

Horizontal motion:

$$v = \frac{x}{t} \quad \Longrightarrow \quad 1600\cos 55 = \frac{x}{267.47\ldots} \quad \Longrightarrow \quad \boldsymbol{x = 245000m\ to\ 3\ s.f.}$$

3. Calculate the maximum height reached by the shells.

Vertical motion:

$$v^2 = u^2 + 2as \quad \Longrightarrow \quad 0 = (1600\sin 55)^2 + 2(-9.8)h \quad \Longrightarrow \quad \boldsymbol{h = 87600m\ to\ 3\ s.f.}$$

4. Compare your results to the observational data below, and comment on any discrepancy. In particular, discuss any assumptions that you made and how your model may be refined.

Paris gun statistics:

Flight Time	Range	Maximum Altitude
$182\ seconds$	$130\ km$	$42.3\ km$

The gun's actual time of flight was only around $\frac{2}{3}$ of that calculated, and the range not much more than $\frac{1}{2}$ of our value. The altitude was even less than $\frac{1}{2}$ of our value.

These large discrepancies can be attributed primarily to **air resistance**. A shell launched at such enormous speeds would have a huge amount of air resistance. While this can't be modelled using constant acceleration equations, it can be taken into account, and was reduced somewhat in practice by increasing the angle from the usual $45°$ to $55°$ in order for the shell to reach heights where air pressure would be significantly lower. Another factor the gunners had to take into account was the curvature of the earth. While for most artillery purposes, this would be so minor as to not affect calculations, for the Paris gun it was necessary to consider the difference in height due to this curvature. Note finally that while 40 kilometres above the surface is significant in terms of air resistance, it only reduces apparent gravity by around 1% (and only for the highest part of the motion), so the impact of this – especially when compared to air resistance – is negligible.