

Gun-running and Numerical Methods

For finding the roots of an equation, we can use:

Bisection

Linear Interpolation

The Newton-Raphson Method

Each subsequent method requires more information and more calculation, but generally yields better results more quickly.

They may be compared to a gun-runner trying to slip over the US-Mexico border:



Bisection – go halfway between the two points

What we need:	<p><i>Thanks to some careless tweeting on the part of our gun-runner, we know that he was in the US at 12 noon, and that by 8pm he was in Mexico.</i></p> <p><i>With no other information, our best guess for when he crossed the border would be the halfway point, 4pm.</i></p>
Two x values either side of the root, and $f(x)$	
What we calculate: The sign (positive/negative) of $f(x)$ for each x value The halfway point of the two x values	

Linear Interpolation – assume constant gradient between two points

What we need:	<p><i>The use of a credit card at a drive-thru tells us our gun-runner stopped for coffee in San Antonio, 200 miles north of the border at 11am, and was sighted in a cafe in Monterrey, 100 miles south of the border, at 5pm.</i></p> <p><i>Assuming a constant speed, our best guess for when he crossed the border would be 3pm.</i></p>
Two x values either side of the root, and $f(x)$	
What we calculate: The value of $f(x)$ for each x value The equation of the straight line connecting these two coordinates The point at which this line crosses the x -axis	

The Newton-Raphson Method – extrapolate from gradient at given point

What we need:	<p><i>Our gun-runner's vehicle was sighted by the Feds going 60mph on I-35, 25 miles north of the border at 2:50pm.</i></p> <p><i>Assuming his speed remains the same, our best guess for when he crossed the border would be 3:15pm.</i></p>
A single x value, $f(x)$ and $f'(x)$	
What we calculate: The values of $f(x)$ and $f'(x)$ The equation of the straight line passing through $(x, f(x))$ with gradient $f'(x)$. The point at which this line crosses the x -axis	

Note: Formulae are readily derivable for each of these methods, but it is important to become familiar with the concepts first and foremost, and learn not to depend on formulae alone for solving problems.