Ships that pass in the night

2-D vectors & optimisation – M1 and C1

In 1845 Lord Franklin took two ships and more than 100 men on an expedition to find a Northwest Passage around the pole. HMS Erebus and HMS Terror were both equipped with state-of-the-art steam engines, allowing them to travel at 7.4 km/h.

At 0300 hours the Terror is travelling with a constant velocity of $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$ km/h.

The Erebus, which is 9km north and 1.5km east of the Terror, is travelling with a constant velocity of $\begin{bmatrix} -7 \\ 1 \end{bmatrix}$ km/h.

1) Calculate the speed and direction of each vessel. Give your answers in km/h and as a 3-figure bearing correct to 1 d.p.

2) Find an expression for the position of each vessel at time *t*.

3) Find an expression for the distance between the two vessels at time *t*, and hence calculate the time when they are closest as well as their distance apart at this point.

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1) Calculate the speed and direction of each vessel. Give your answers in km/h and as a 3-figure bearing correct to 1 d.p.

Terror:

$$Speed = \sqrt{(-4)^2 + 5^2} = 6.40kmh^{-1}$$

$$Bearing = 270 + \tan^{-1}\frac{5}{4} = 321.3°$$

Erebus:

$$Speed = \sqrt{(-7)^2 + 1^2} = 7.07kmh^{-1}$$

$$Bearing = 270 + \tan^{-1}\frac{1}{7} = 278.1°$$

2) Find an expression for the position of each vessel at time *t*.

Terror:

$$x_T = \begin{bmatrix}0\\0\end{bmatrix} + t\begin{bmatrix}-4\\5\end{bmatrix} = \begin{bmatrix}-4t\\5t\end{bmatrix}$$

Erebus:

$$x_E = \begin{bmatrix}1.5\\9\end{bmatrix} + t\begin{bmatrix}-7\\1\end{bmatrix} = \begin{bmatrix}-7t+1.5\\t+9\end{bmatrix}$$

3) Find an expression for the distance between the two vessels at time *t*, and hence calculate the time when they are closest as well as their distance apart at this point.

Displacement:

$$x_T - x_E = \begin{bmatrix}3t-1.5\\4t-9\end{bmatrix}$$

Distance:

$$s = |x_T - x_E| = \left|\begin{bmatrix}3t-1.5\\4t-9\end{bmatrix}\right| = \sqrt{(3t-1.5)^2 + (4t-9)^2}$$

Closest when *s* is minimum:

$$\sqrt{(3t-1.5)^2 + (4t-9)^2}\ minimum\ when\ (3t-1.5)^2 + (4t-9)^2\ minimum$$

$$\Rightarrow \quad 25t^2 - 81t + 83.25\ minimum$$

Differentiating gives:

$$50t - 81 = 0 \quad \Rightarrow \quad t = \frac{81}{50} = 1.62\ hours \quad \Rightarrow \quad time = 0437:12$$

Substituting for *s*:

$$s = \sqrt{(3(1.62) - 1.5)^2 + (4(1.62) - 9)^2} = 4.2km$$