Ships that pass in the night

2-D vectors & optimisation – M1 and C1

In 1845 Lord Franklin took two ships and more than 100 men on an expedition to find a Northwest Passage around the pole. HMS Erebus and HMS Terror were both equipped with state-of-the-art steam engines, allowing them to travel at 7.4 km/h.

At 0300 hours the Terror is travelling with a constant velocity of $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$ km/h.

The Erebus, which is 9km north and 1.5km east of the Terror, is travelling with a constant velocity of $\begin{bmatrix} -7 \\ 1 \end{bmatrix}$ km/h.

1) Calculate the speed and direction of each vessel. Give your answers in km/h and as a 3-figure bearing correct to 1 d.p.

2) Find an expression for the position of each vessel at time t.

3) Find an expression for the distance between the two vessels at time t, and hence calculate the time when they are closest as well as their distance apart at this point.

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1) Calculate the speed and direction of each vessel. Give your answers in km/h and as a 3-figure bearing correct to 1 d.p.

Terror:

$$Speed = \sqrt{(-4)^2 + 5^2} = 6.40kmh^{-1}$$

$$Bearing = 270 + \tan^{-1}\frac{5}{4} = 321.3°$$

Erebus:

$$Speed = \sqrt{(-7)^2 + 1^2} = 7.07kmh^{-1}$$

$$Bearing = 270 + \tan^{-1}\frac{1}{7} = 278.1°$$

2) Find an expression for the position of each vessel at time *t*.

Terror:

$$x_T = \begin{bmatrix}0\\0\end{bmatrix} + t\begin{bmatrix}-4\\5\end{bmatrix} = \begin{bmatrix}-4t\\5t\end{bmatrix}$$

Erebus:

$$x_E = \begin{bmatrix}1.5\\9\end{bmatrix} + t\begin{bmatrix}-7\\1\end{bmatrix} = \begin{bmatrix}-7t+1.5\\t+9\end{bmatrix}$$

3) Find an expression for the distance between the two vessels at time *t*, and hence calculate the time when they are closest as well as their distance apart at this point.

Displacement:

$$x_T - x_E = \begin{bmatrix}3t-1.5\\4t-9\end{bmatrix}$$

Distance:

$$s = |x_T - x_E| = \left\|\begin{bmatrix}3t-1.5\\4t-9\end{bmatrix}\right\| = \sqrt{(3t-1.5)^2 + (4t-9)^2}$$

Closest when *s* is minimum:

$$\sqrt{(3t-1.5)^2 + (4t-9)^2}\ minimum\ when\ (3t-1.5)^2 + (4t-9)^2\ minimum$$

$$\Rightarrow \quad 25t^2 - 81t + 83.25\ minimum$$

Differentiating gives:

$$50t - 81 = 0 \quad \Rightarrow \quad t = \frac{81}{50} = 1.62\ hours \quad \Rightarrow \quad time = 0437:12$$

Substituting for *s*:

$$s = \sqrt{(3(1.62) - 1.5)^2 + (4(1.62) - 9)^2} = 4.2km$$