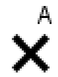
Introduction to Loci

The locus ('position') of points that fit a rule shows where the points are allowed to be.

<u>Investigation 1: Fixed distance from a point</u>

Using a ruler, **mark a point** with a × exactly $3cm$ from the point A (in any direction).

Now **mark 4 more points**, all exactly $3cm$ from the point A, and all in different directions.



Finally, use your compass to construct a **circle of radius $3cm$ with A at the centre**.

This shows *all* the points $3cm$ from A:

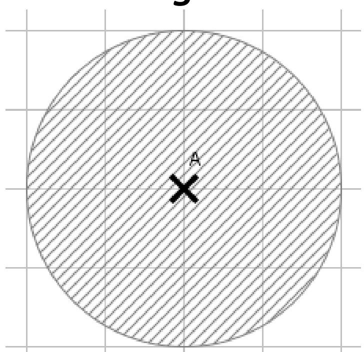
"The locus of points $3cm$ from A"

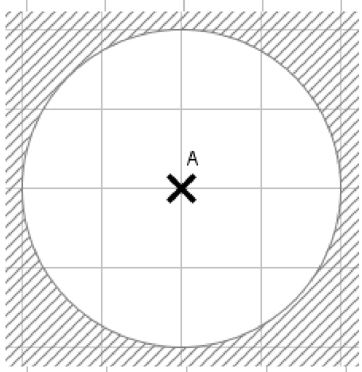
> ***Key Idea:*** **All points a fixed distance from A form a *circle* around A.**

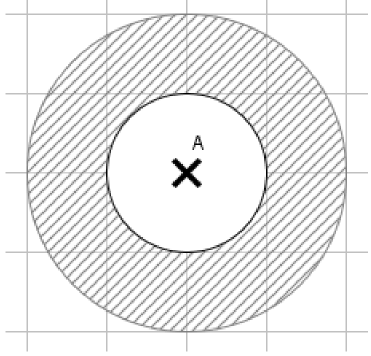
<u>Investigation 2: Max or min distance from a point</u>

Draw arrows to correctly match the shaded area of each diagram to the right description:

Locus Diagrams:







Locus Descriptions:

"The locus of points ***more than $2cm$*** from A."

"The locus of points ***more than $1cm$*** from A, but ***also*** ***less than $2cm$*** from A."

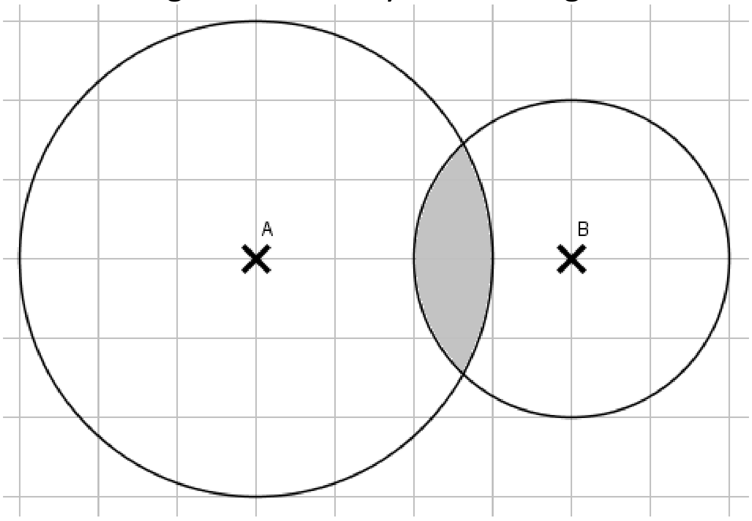
"The locus of points ***less than $2cm$*** from A."

> **Key Idea: All points *less than* a distance from A form a *shaded circle* around A.**
> All points *more than* a certain distance from A can be shown by shading *outside* the circle.

Investigation 3: Combining loci

Describe the locus of the shaded regions shown by underlining the correct phrases:

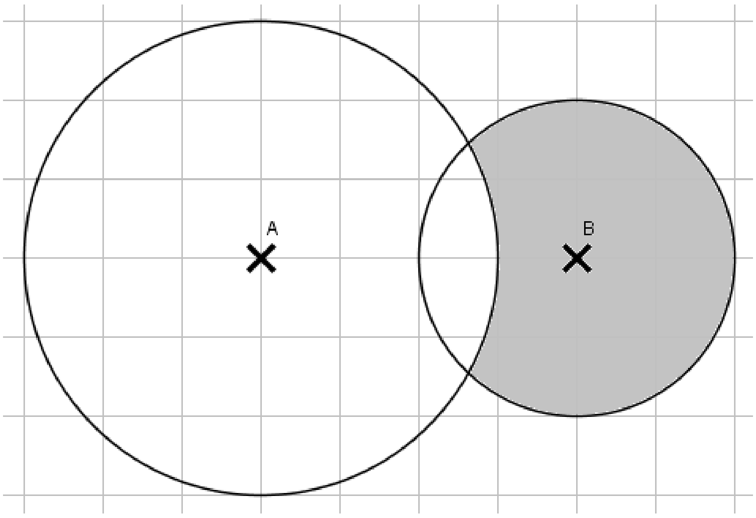
a)



The shaded part is:

less than / more than $3\ cm$ from A, *and* **less than / more than** $2\ cm$ from B.

b)

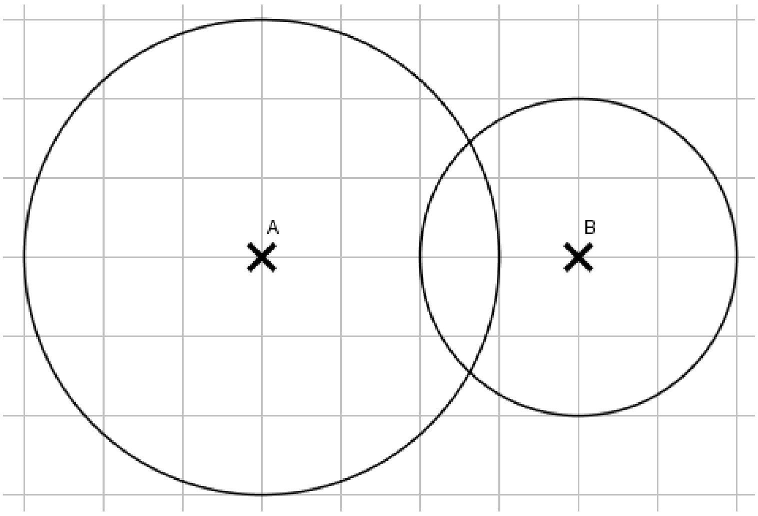


The shaded part is:

less than / more than $3\ cm$ from A, *and* **less than / more than** $2\ cm$ from B.

c) Shade in the region on the diagram below which fits the following description:

"The locus of points **more than 3 cm from $\boldsymbol{A}$** *and* **more than $\boldsymbol{2cm}$ from $\boldsymbol{B}$**"

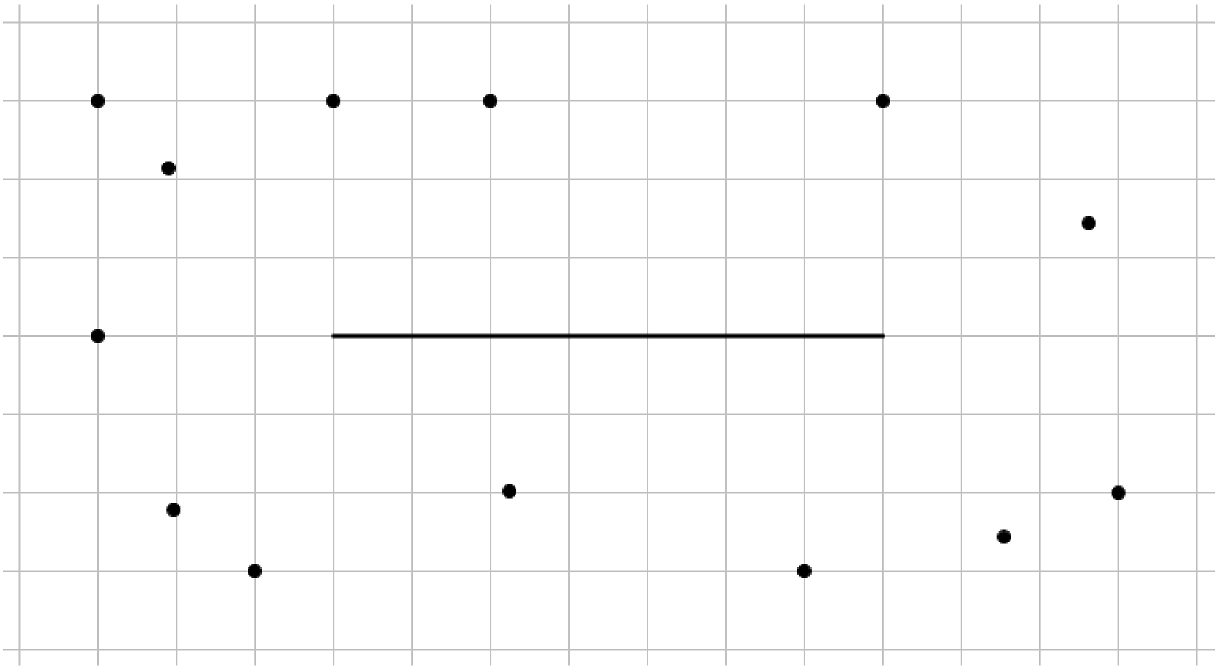


Key Idea: Loci can be combined by finding the points that fit *every* rule given.

This could be just a point or a few points, a line or curve, or even an entire shaded region.

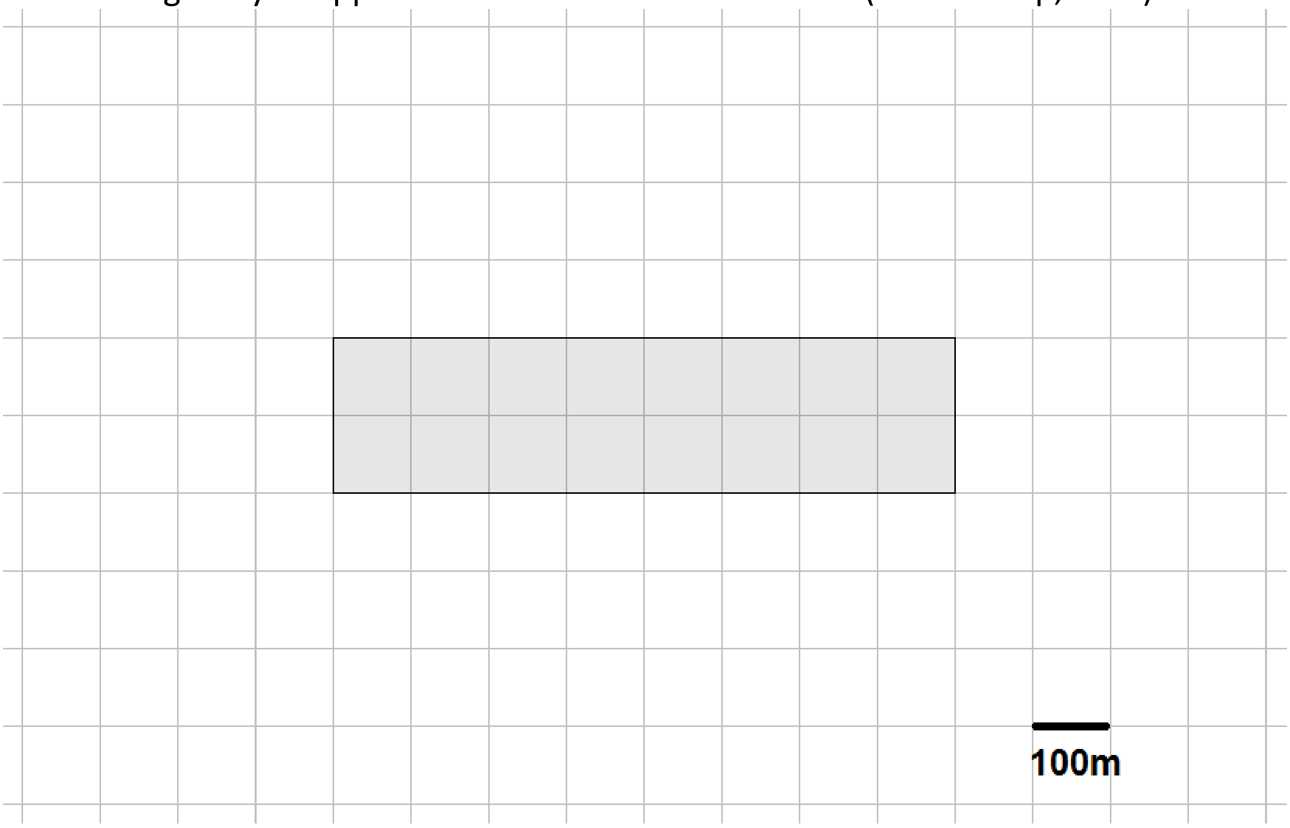
<u>Investigation 4: Fixed distance from a line or a rectangle</u>

1. The points marked on the diagram below are **supposed** to be **exactly 3cm from the line**. **Four** of them are **not in the right position**.
By measuring accurately with a ruler, find these four wrong points, and **cross them out**.



Next, with a ruler and compass, construct the locus of points **exactly $3cm$ from the line**.
Remember the distance you measure should be the most direct route to the line.

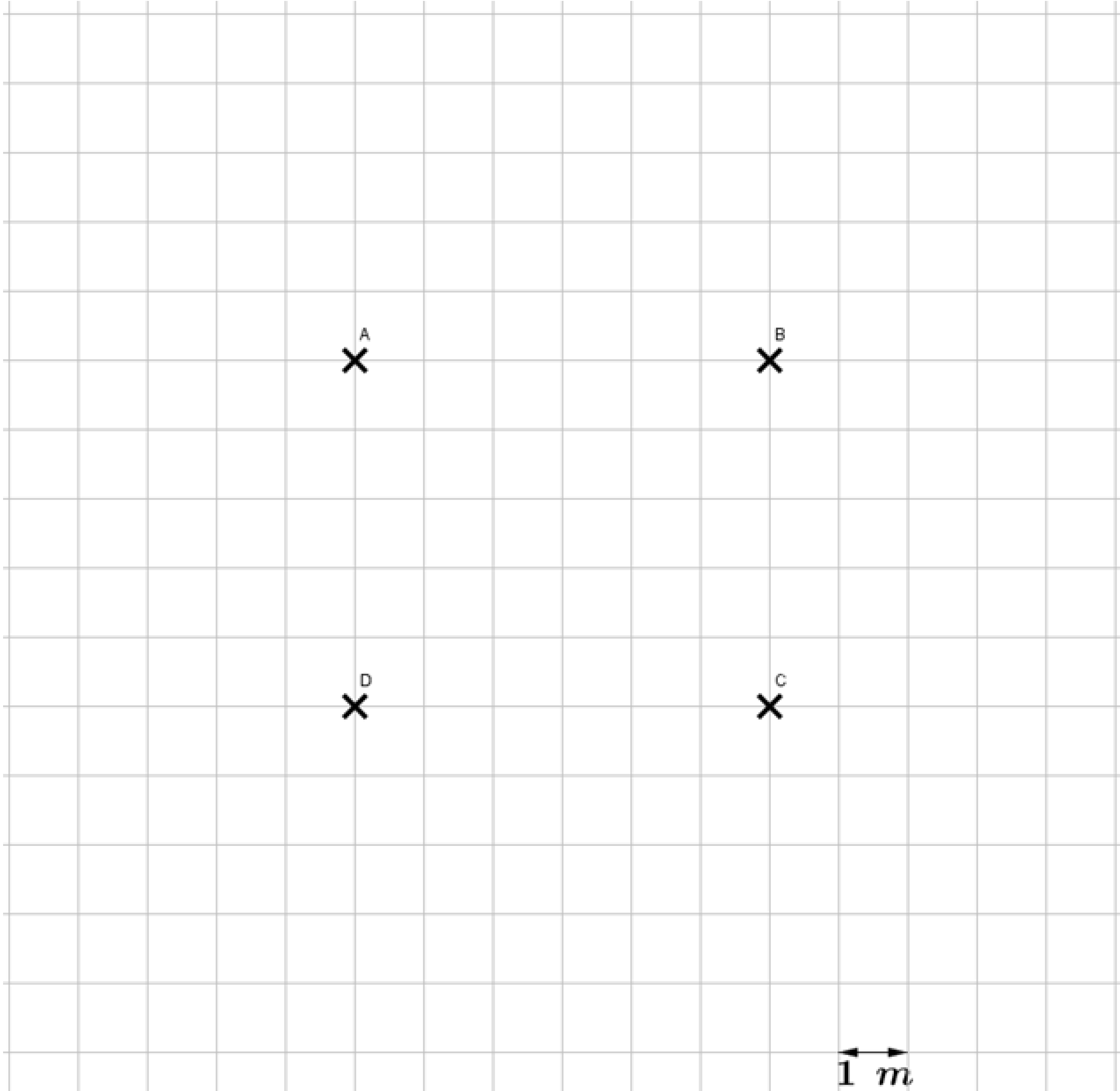
2. The rectangle below represents the perimeter fence of a military compound.
You are in danger if you approach within $300m$ of the fence (on this map, $3cm$).



Construct the locus of points **exactly $3cm$ from the rectangle** to show the danger zone.
Remember that this will be a straight line parallel to the fences, but curved around corners.

Floodlit

Four floodlights are positioned in a field as shown in the diagram:



Each floodlight lights up the ground for 5 $metres$ in every direction.

1. Construct the locus of points **exactly 5 metres** from each of the four points.
The locus of points $5cm$ from a point is a circle with radius $5cm$.

2. In each region, write down the number of different lights you would be lit up by.
For instance, if you are within 5 metres of both A and C (but not B or D) write 2.

3. Colour-code these regions, and make a key to show what each code represents.
If you don't have colours, use different patterns of shading or hatching.

Equidistance

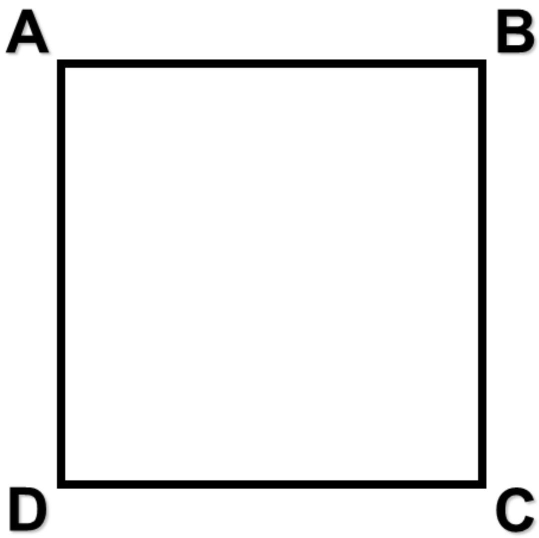
***Locus** means 'place' or 'position'.* ***Equidistant** means 'equal distance'.*

Task A

1. Mark a point (×) *anywhere* in the square which is *exactly* the same distance from the top (AB) as it is from the bottom (DC).

2. Mark two more points somewhere else, also the same distance from AB as DC.

3. Draw a *line* that goes through every possible point which is equidistant (the same distance) from AB as DC.



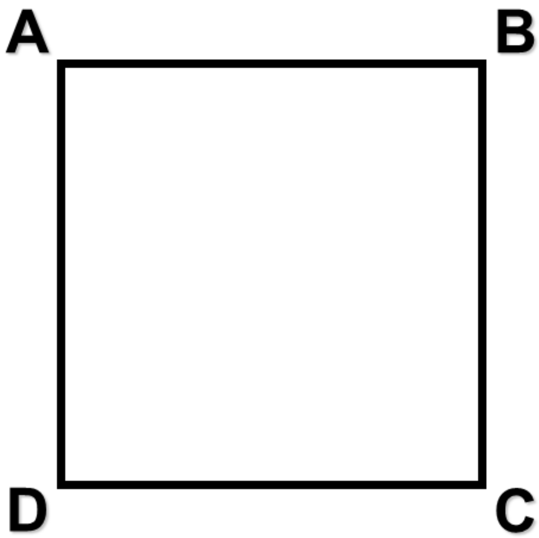
You have constructed **"The locus of points in the square equidistant from AB and DC"**.

Task B

1. Mark a point (×) *anywhere* in the square which is *exactly* the same distance from the point A as it is from the point C.

2. Mark two more points somewhere else, also the same distance from A as C.
Hint: your points don't have to be in line with A and C. Might B or D work?

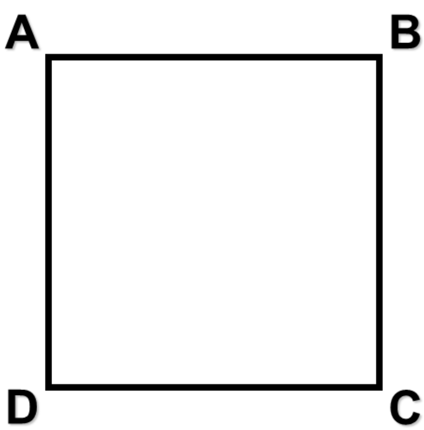
3. Draw a *line* that goes through every possible point which is equidistant (the same distance) from A as C.



You have constructed **"The locus of points in the square equidistant from A and C"**.

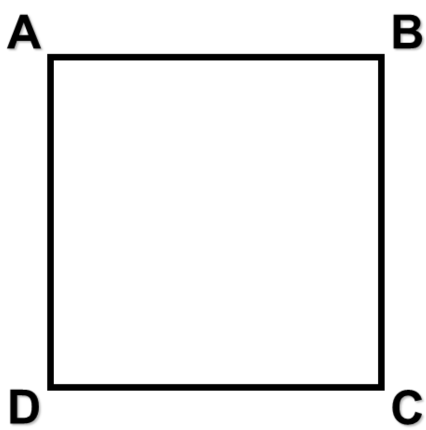
Extension Tasks

1.



Shade in the parts of the square which are:
Closer to AD than BC *and*
closer to DC than AB.

2.



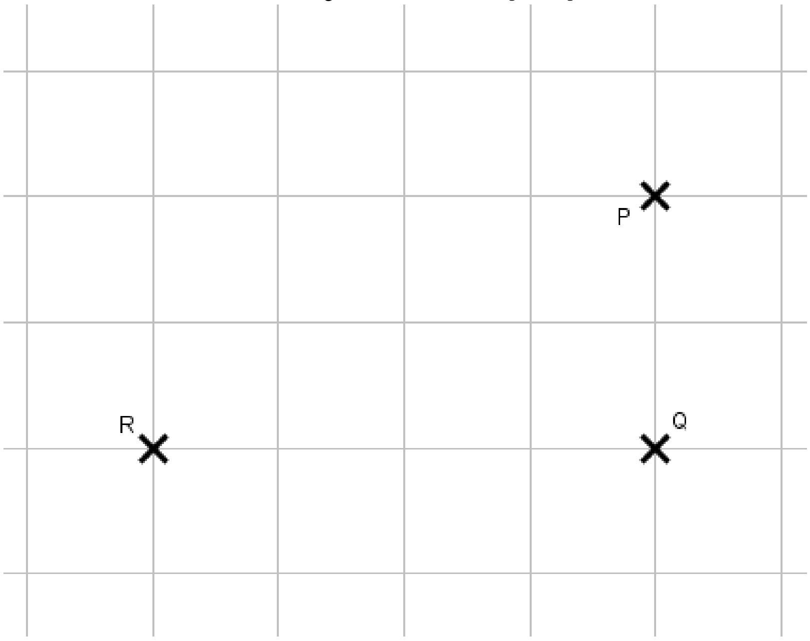
Shade in the parts of the square which are:
Closer to A than C *and*
closer to B than D.

Hidden Circles

A circle through three points can be found using *perpendicular bisectors*.

Section A

1. Draw the **horizontal line** exactly halfway between P and Q.
This line is ***equidistant*** from P and Q. It is the ***perpendicular bisector*** of PQ.



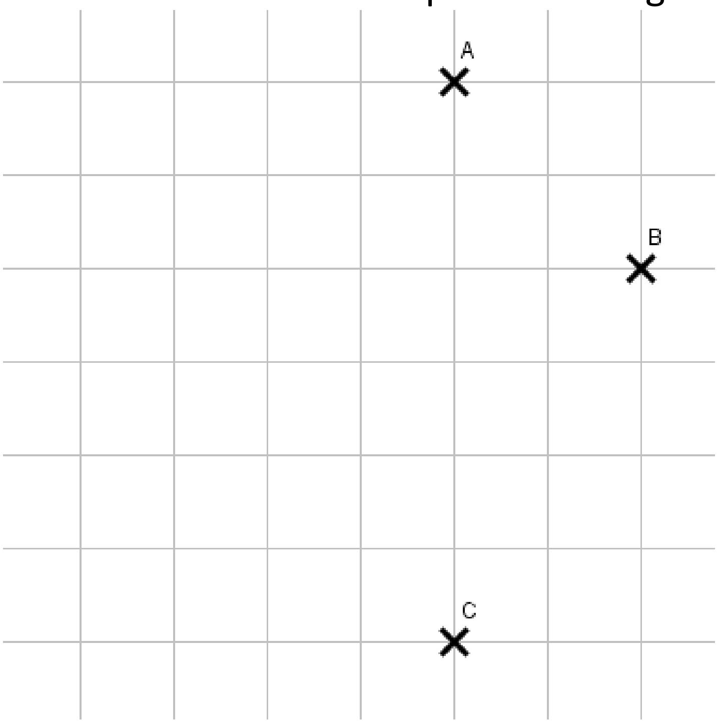
2. Draw the **vertical line** exactly halfway between Q and R.
This line is ***equidistant*** from Q and R. It is the ***perpendicular bisector*** of QR.

3. Mark with a × the point where your two lines cross.
This will be the *centre* of your circle.

4. Set your compass so it reaches from your point (the centre) to P, and draw the circle. *You should find that this circle passes through P, Q and R.*

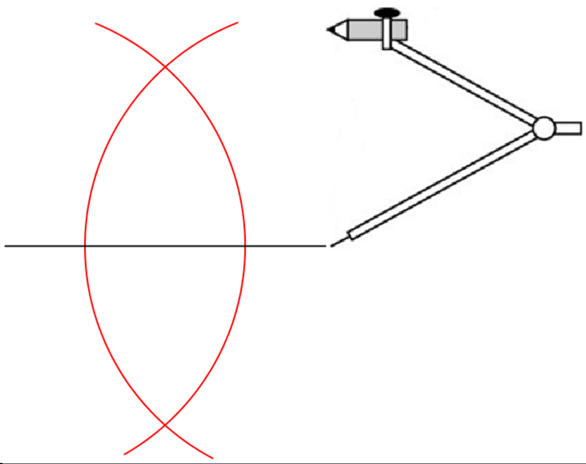
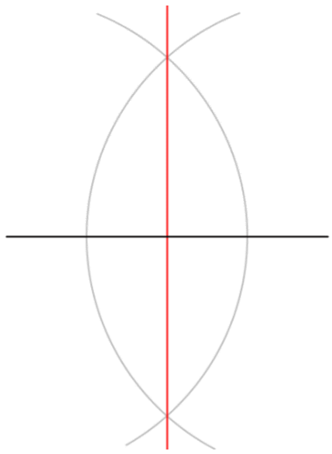
Section B

Use the same method to find the circle that passes through A, B and C:



Recap: Perpendicular Bisector

To find the perpendicular bisector of a line segment:

1. Set the compass radius to over half the length of the line, and make an arc from each end. 	2. Draw a straight line through both crossing points. This line is the perpendicular bisector. 

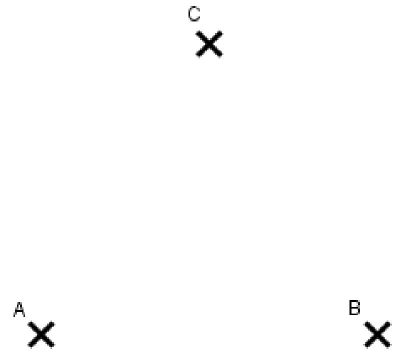
Note: by treating two points as the ends of a line segment, you can construct the perpendicular bisector without even drawing the line segment between.

Section C

Draw a line segment between any two of the three points below.
Construct the perpendicular bisector of this line segment.
Repeat the process for a different pair of points.
The centre of our circle is the point where the two perpendicular bisectors cross, and the radius is the distance from here to any of the three points.

Signal Tower Loci

The diagram below shows the position of three mobile phone signal towers, all $4km$ apart.



The map above is scaled so $1cm$ on the map represents $1km$ in reality.

1. The most powerful signal tower is A, which can detect a phone up to $3km$ away.
   Construct the locus of points *exactly* $3cm$ from A on the diagram.

2. Signal tower B can detect a phone up to $2km$ away. Tower C has a range of $2.5km$.
   Indicate both of these limits on the diagram.

3. Dave's mobile phone is detected by towers A and B, but not C.
   Label with a D the region Dave must be in.

4. Edith's phone is detected by all three towers.
   Label with an E the region Edith must be in.

5. Fred's phone is detected by tower B, but not A or C.
   Label with an F the region Fred must be in.

*6. Tower A is being upgraded.
How large would its new range need to be to make the other two towers obsolete?
(In other words, until the range of A completely covers the range of B and C).

Introduction to Loci SOLUTIONS

The locus ('position') of points that fit a rule shows where the points are allowed to be.

Investigation 1: Fixed distance from a point

Using a ruler, **mark a point** with a × exactly $3cm$ from the point A (in any direction).

Now **mark 4 more points**, all exactly $3cm$ from the point A, and all in different directions.

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This shows *all* the points $3cm$ from A:
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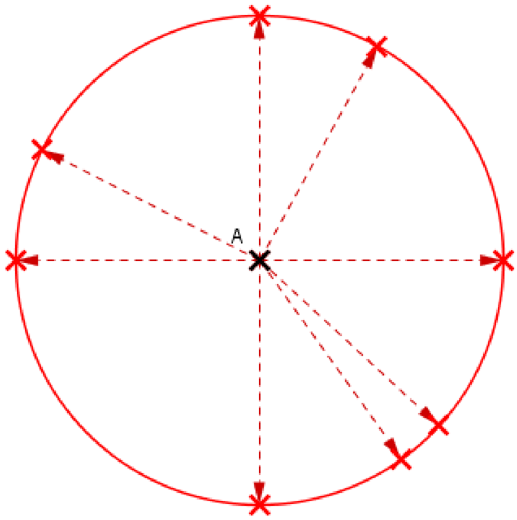

Key Idea: **All points a fixed distance from A form a *circle* around A.**

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Draw arrows to correctly match the shaded area of each diagram to the right description:

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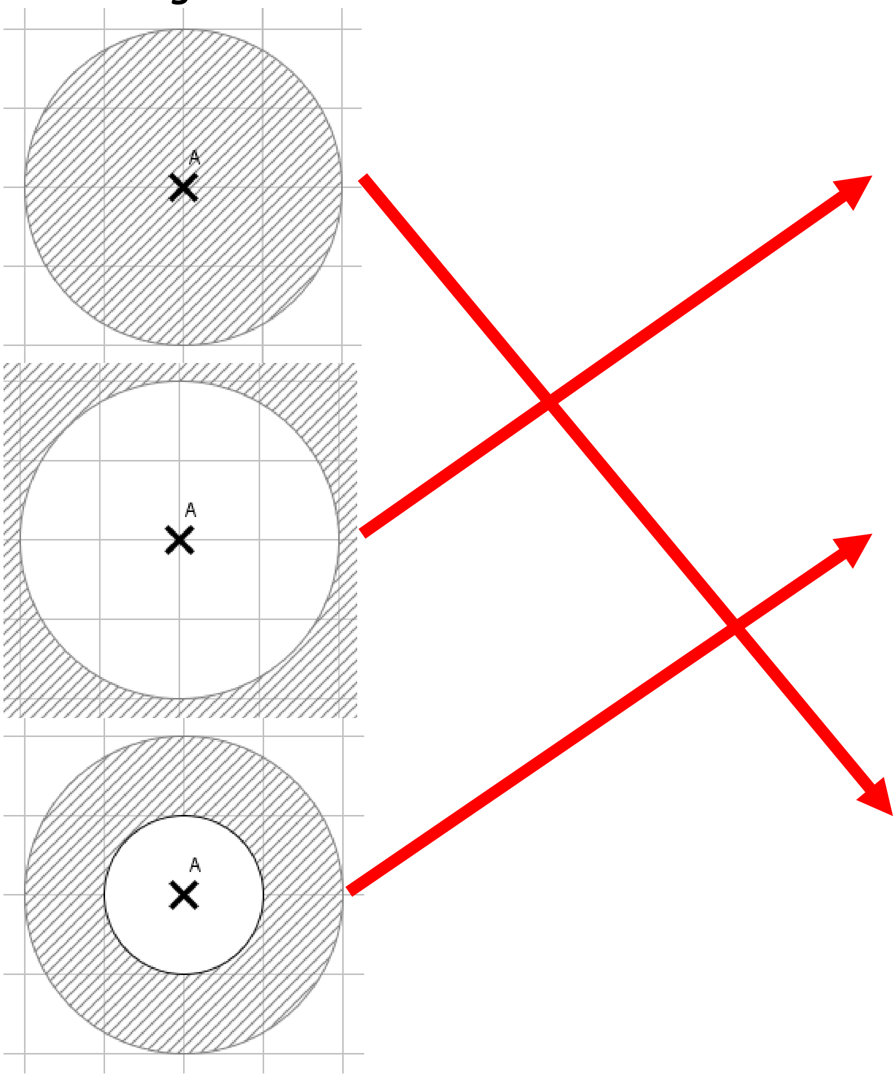

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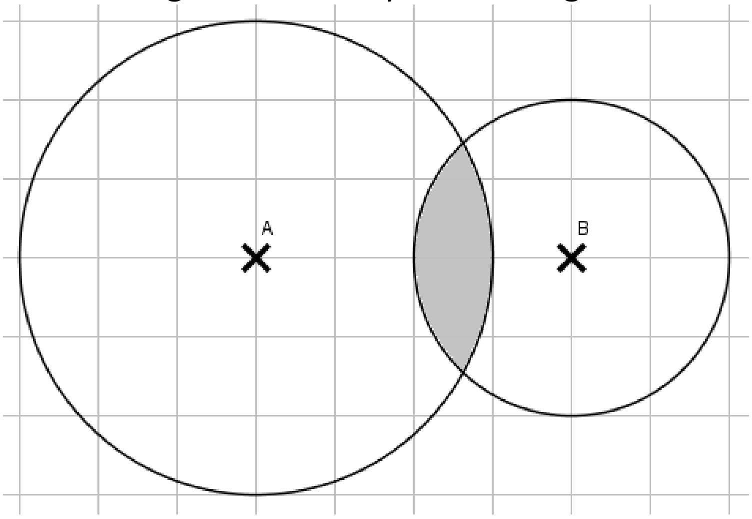

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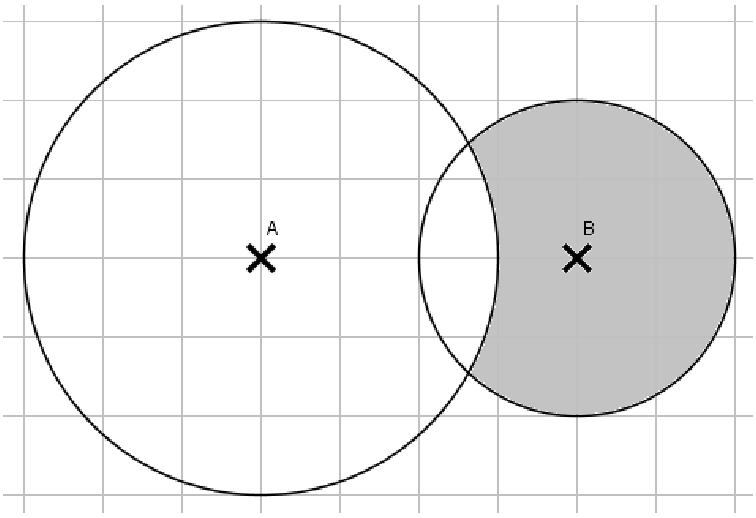

The shaded part is:

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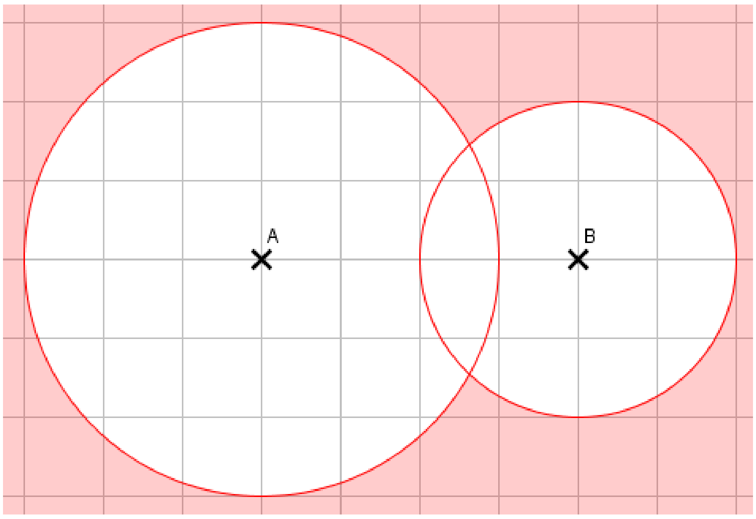

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Investigation 4: Fixed distance from a line or a rectangle

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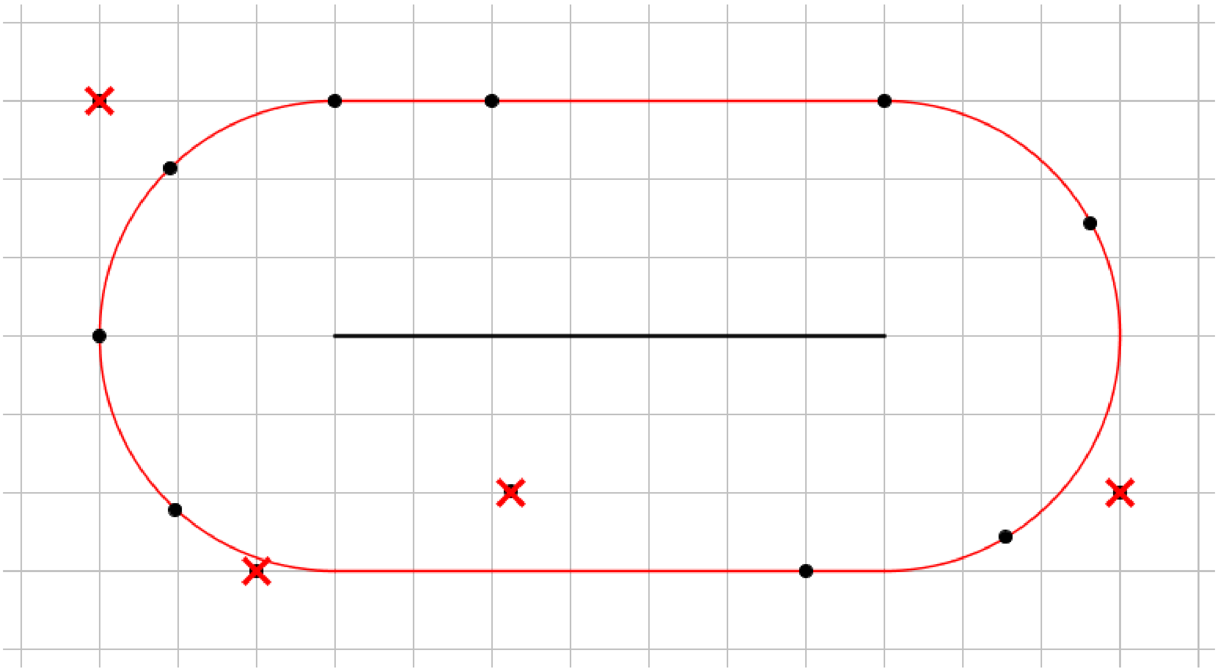

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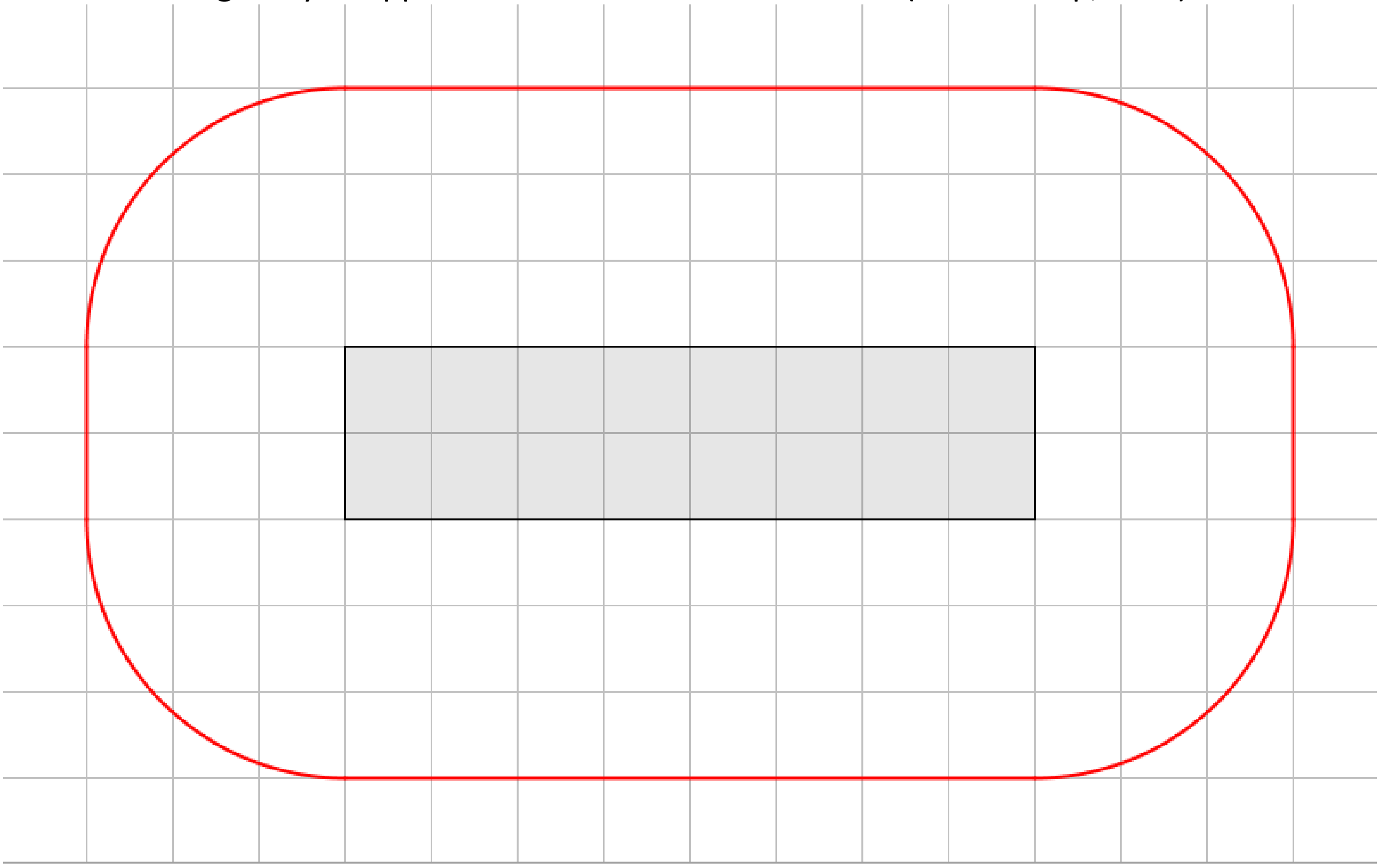
2. The rectangle below represents the perimeter fence of a military compound.
You are in danger if you approach within $300m$ of the fence (on this map, $3cm$).

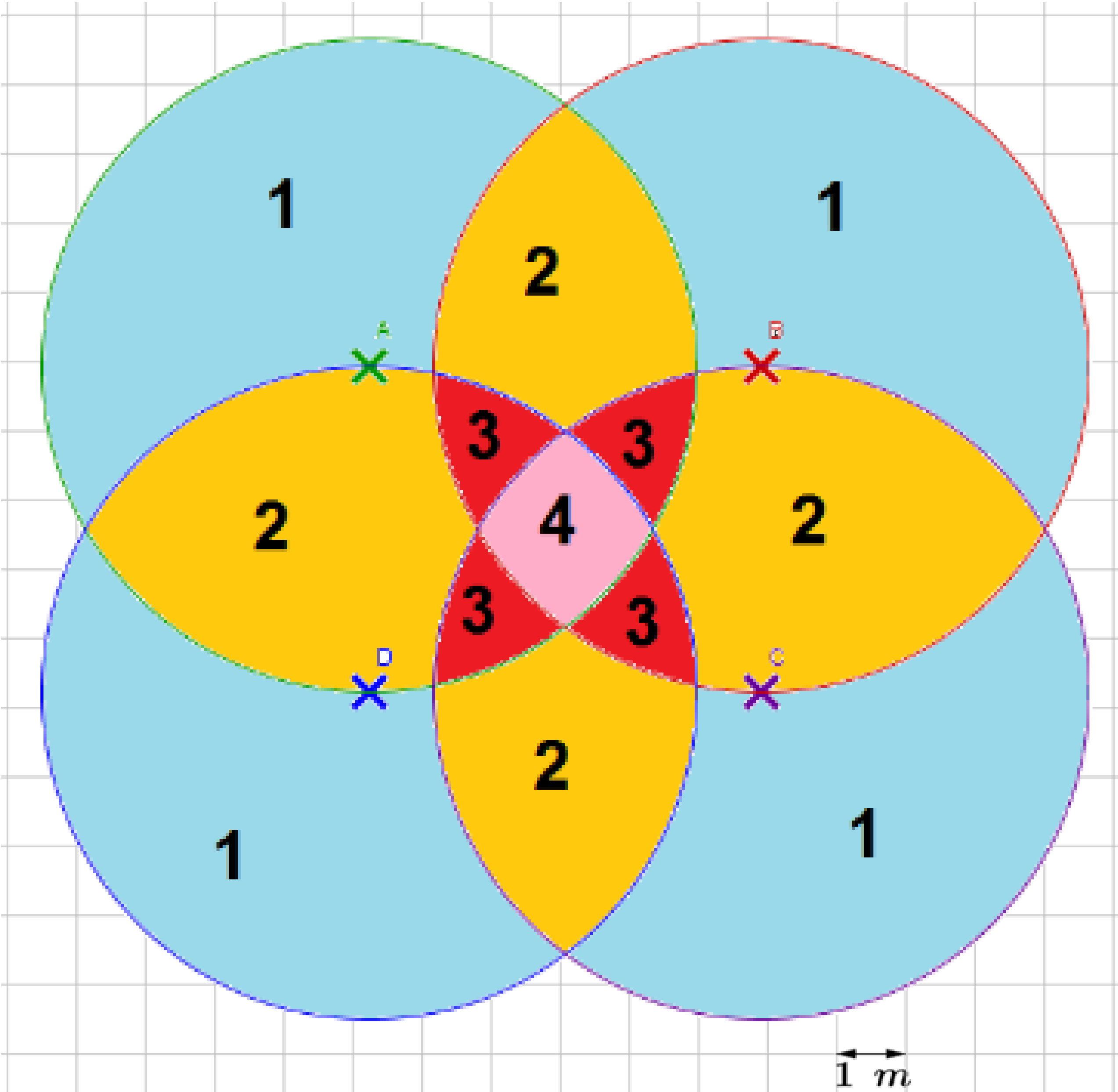


Construct the locus of points **exactly $3cm$ from the rectangle** to show the danger zone.

Remember that this will be a straight line parallel to the fences, but curved around corners.

Floodlit SOLUTIONS

Four floodlights are positioned in a field as shown in the diagram:



Each floodlight lights up the ground for 5 $metres$ in every direction.

1. Construct the locus of points **exactly 5 metres** from each of the four points.
The locus of points $5cm$ *from a point is a circle with radius* $5cm$.

2. In each region, write down the number of different lights you would be lit up by.
For instance, if you are within 5 *metres of both* A *and* C *(but not* B *or* D*) write* 2.

3. Colour-code these regions, and make a key to show what each code represents.
If you don't have colours, use different patterns of shading or hatching.

Equidistance SOLUTIONS

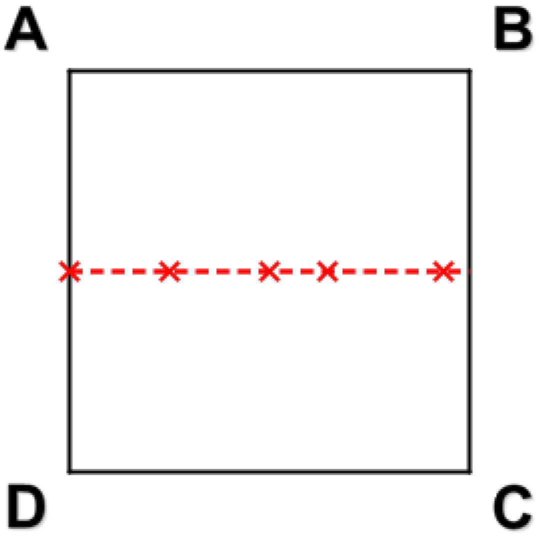
***Locus** means 'place' or 'position'.* ***Equidistant** means 'equal distance'.*

Task A

1. Mark a point (×) *anywhere* in the square which is *exactly* the same distance from the top (AB) as it is from the bottom (DC).

2. Mark two more points somewhere else, also the same distance from AB as DC.

3. Draw a *line* that goes through every possible point which is equidistant (the same distance) from AB as DC.



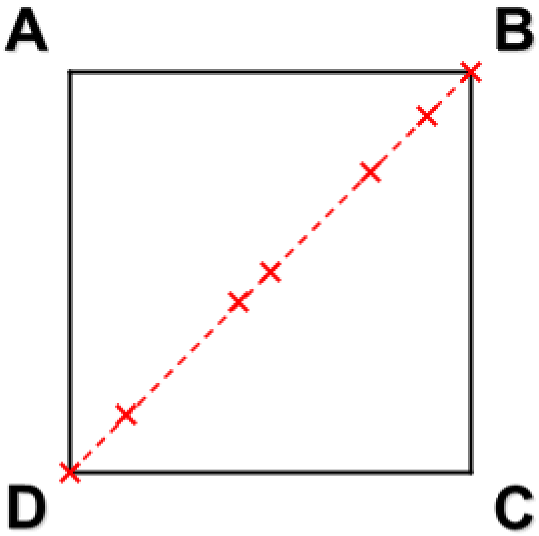
You have constructed **"The locus of points in the square equidistant from AB and DC"**.

Task B

1. Mark a point (×) *anywhere* in the square which is *exactly* the same distance from the point A as it is from the point C.

2. Mark two more points somewhere else, also the same distance from A as C.
Hint: your points don't have to be in line with A and C. Might B or D work?

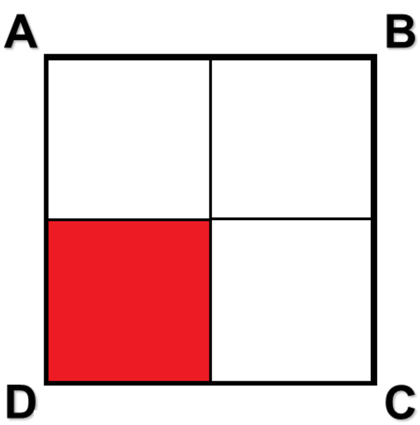
3. Draw a *line* that goes through every possible point which is equidistant (the same distance) from A as C.



You have constructed **"The locus of points in the square equidistant from A and C"**.

Extension Tasks

1.



Shade in the parts of the square which are:
Closer to AD than BC *and*
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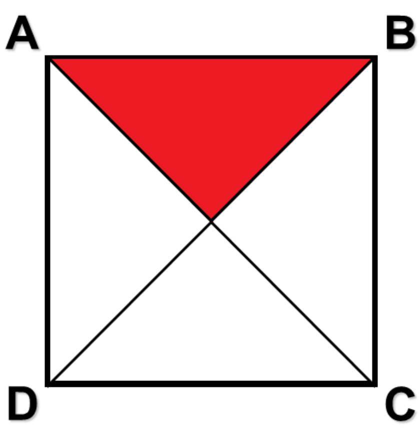


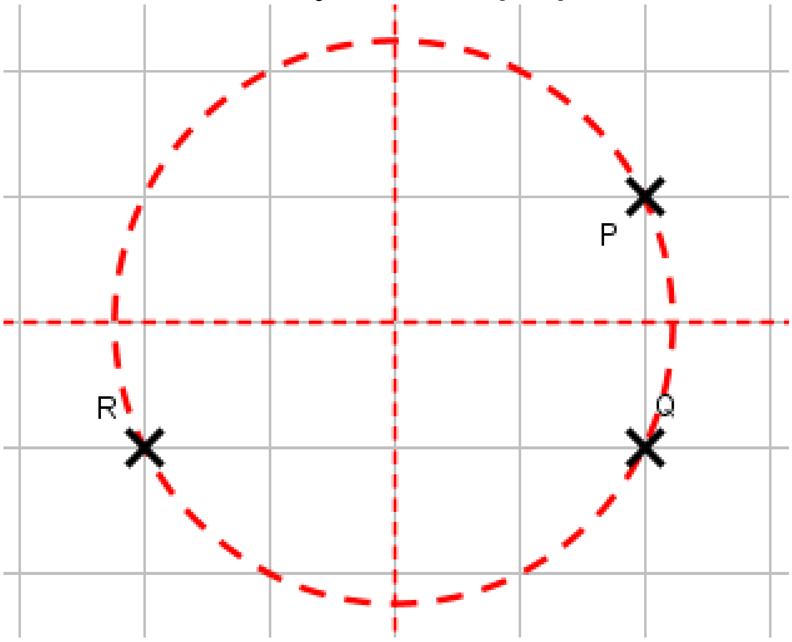
Shade in the parts of the square which are:
Closer to A than C *and*
closer to B than D.

Hidden Circles SOLUTIONS

A circle through three points can be found using *perpendicular bisectors*.

Section A

1. Draw the **horizontal line** exactly halfway between P and Q.
This line is ***equidistant*** from P and Q. It is the ***perpendicular bisector*** of PQ.



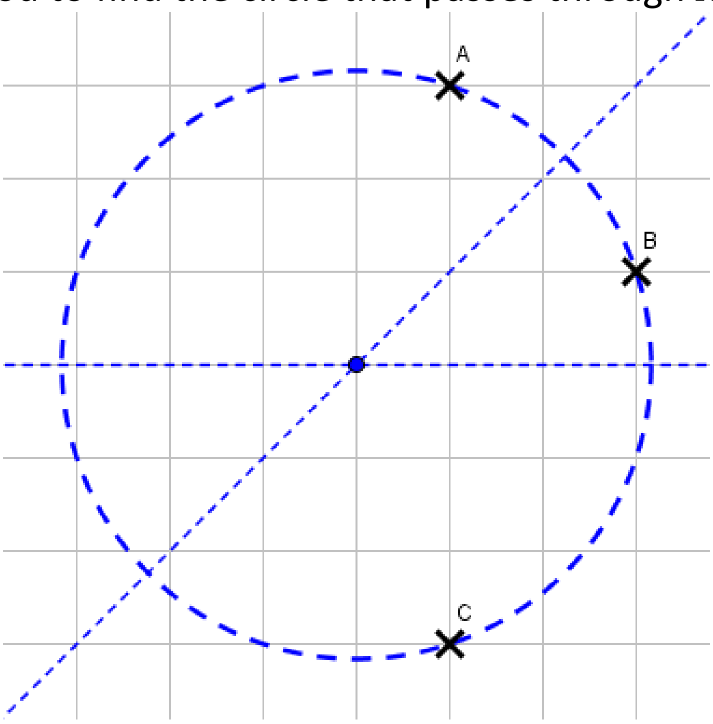
2. Draw the **vertical line** exactly halfway between Q and R.
This line is ***equidistant*** from Q and R. It is the ***perpendicular bisector*** of QR.

3. Mark with a $\times$ the point where your two lines cross.
This will be the *centre* of your circle.

4. Set your compass so it reaches from your point (the centre) to P, and draw the circle. *You should find that this circle passes through P, Q and R.*

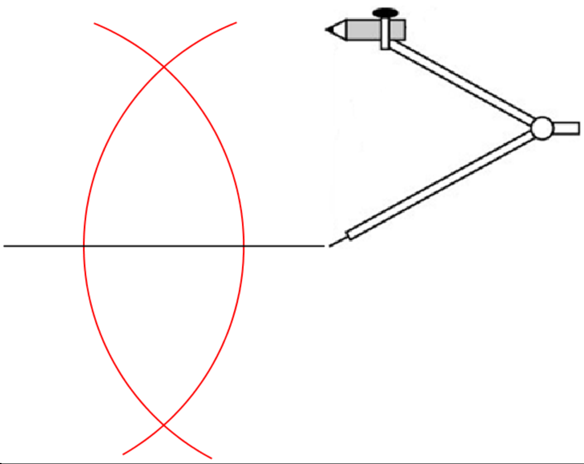
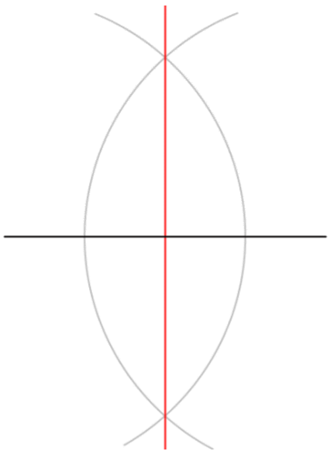
Section B

Use the same method to find the circle that passes through A, B and C:



Recap: Perpendicular Bisector

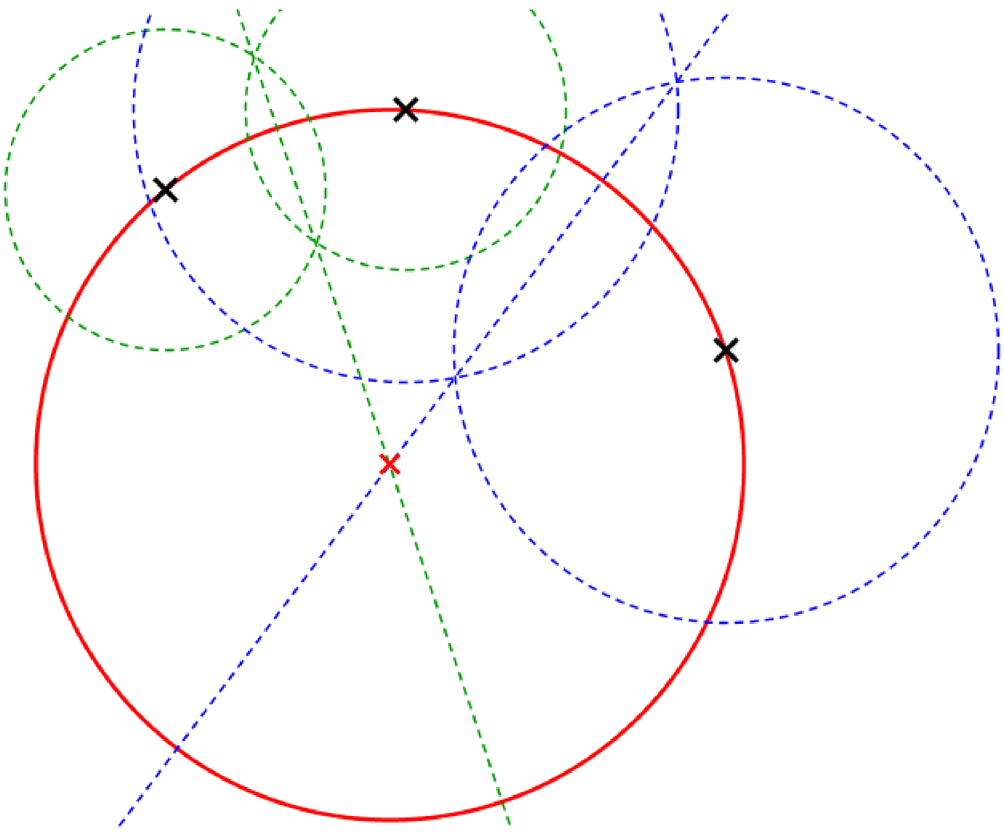
To find the perpendicular bisector of a line segment:

1. Set the compass radius to over half the length of the line, and make an arc from each end. 	2. Draw a straight line through both crossing points. This line is the perpendicular bisector. 

Note: by treating two points as the ends of a line segment, you can construct the perpendicular bisector without even drawing the line segment between.

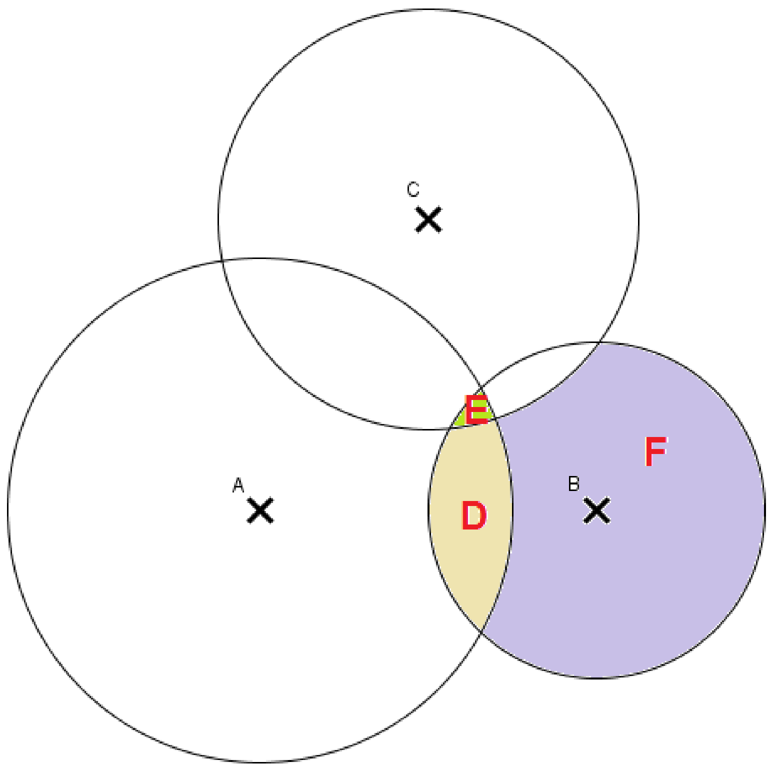
Section C

Draw a line segment between any two of the three points below.
Construct the perpendicular bisector of this line segment.
Repeat the process for a different pair of points.
The centre of our circle is the point where the two perpendicular bisectors cross, and the radius is the distance from here to any of the three points.



Signal Tower Loci SOLUTIONS

The diagram below shows the position of three mobile phone signal towers, all $4km$ apart.



The map above is scaled so $1cm$ on the map represents $1km$ in reality.

1. The most powerful signal tower is A, which can detect a phone up to $3km$ away.
   Construct the locus of points *exactly* $3cm$ from A on the diagram.

   Circle of radius $3cm$ centred at A

2. Signal tower B can detect a phone up to $2km$ away. Tower C has a range of $2.5km$.
   Indicate both of these limits on the diagram.

   Circle of radius $2cm$ centred at B, and a circle of radius $2.5cm$ centred at C

3. Dave's mobile phone is detected by towers A and B, but not C.
   Label with a D the region Dave must be in.

   In the overlap of A's and B's circles, but not in the overlap of all three.

4. Edith's phone is detected by all three towers.
   Label with an E the region Edith must be in.

   In the overlap of all three circles.

5. Fred's phone is detected by tower B, but not A or C.
   Label with an F the region Fred must be in.

   In B's circle, but not in either of the other two.

*6. Tower A is being upgraded.

How large would its new range need to be to make the other two towers obsolete?

$\mathbf{6.5km}$. The most distant point to be covered is on the edge of C's range. C is $4km$ from A, and has a range which reaches a further $2.5km$. $4 + 2.5 = \mathbf{6.5km}$.