Three-sided Inequalities

Some three-sided inequalities can be solved just like a normal two-sided inequality.

Eg:

$-1 < 3x + 8 \leq 11$

$-9 < 3x \leq 3$

$\boldsymbol{-3 < x \leq 1}$

But if x appears in more than one section, it is usually best to treat the inequality as two separate simultaneous inequalities (solve each independently, then identify the range of values that satisfy both). For instance, using the same example as on the left:

$$-1 < 3x + 8 \leq 11$$

$-1 < 3x + 8$ | $3x + 8 \leq 11$

$-9 < 3x$ | $3x \leq 3$

$-3 < x$ | $x \leq 1$

$\rightarrow \quad -3 < x \ and \ x \leq 1 \quad \leftarrow$

$$\boldsymbol{-3 < x \leq 1}$$

Note: the first method is more efficient in most cases, but the second can be used for a wider range of problems.

1. a) Find the range of values of x which satisfy: $3x - 6 < x + 4$

b) Find the range of values of x which satisfy: $-x < 3x - 6$

c) **Hence**, write down the range of values of x which satisfy: $-x < 3x - 6 < x + 4$

2. Find the range of values of x which satisfy: $-x \geq x + 4 \geq 3x - 6$

3. Find the range of values of x which satisfy: $3x - 6 \leq -x < x + 4$

4. Find the range of values of x which satisfy: $-x < x + 4 \leq 3x - 6$

5. a) Find the range of values of x which satisfy: $x + 4 < -x$

b) Find the range of values of x which satisfy: $-x < 3x - 6$

c) **Hence**, write down the range of values of x which satisfy: $x + 4 < -x < 3x - 6$

Three-sided Inequalities SOLUTIONS

1. a) Find the range of values of x which satisfy: $3x - 6 < x + 4$

$$2x - 6 < 4$$
$$2x < 10$$
$$\boldsymbol{x < 5}$$

b) Find the range of values of x which satisfy: $-x < 3x - 6$

$$0 < 4x - 6$$
$$6 < 4x$$
$$1.5 < x \quad or \quad \boldsymbol{x > 1.5}$$

c) **Hence**, write down the range of values of x which satisfy: $-x < 3x - 6 < x + 4$

$$x < 5 \quad and \quad x > 1.5 \quad \Rightarrow \quad \boldsymbol{1.5 < x < 5}$$

2. Find the range of values of x which satisfy: $-x \geq x + 4 \geq 3x - 6$

$$-x \geq x + 4 \quad \Rightarrow \quad x \leq -2$$
$$x + 4 \geq 3x - 6 \quad \Rightarrow \quad x \leq 5$$
$$x \leq -2 \quad and \quad x \leq 5 \quad \Rightarrow \quad \boldsymbol{x \leq -2}$$

3. Find the range of values of x which satisfy: $3x - 6 \leq -x < x + 4$

$$3x - 6 \leq -x \quad \Rightarrow \quad x \leq 1.5$$
$$-x < x + 4 \quad \Rightarrow \quad x > -2$$
$$x \leq 1.5 \quad and \quad x > -2 \quad \Rightarrow \quad \boldsymbol{-2 < x < 1.5}$$

4. Find the range of values of x which satisfy: $-x < x + 4 \leq 3x - 6$

$$-x < x + 4 \quad \Rightarrow \quad x > -2$$
$$x + 4 \leq 3x - 6 \quad \Rightarrow \quad x \geq 5$$
$$x > -2 \quad and \quad x \geq 5 \quad \Rightarrow \quad \boldsymbol{x \geq 5}$$

5. a) Find the range of values of x which satisfy: $x + 4 < -x$

$$x + 4 < -x$$
$$2x + 4 < 0$$
$$2x < -4$$
$$\boldsymbol{x < -2}$$

b) Find the range of values of x which satisfy: $-x < 3x - 6$

$$-x < 3x - 6$$
$$0 < 4x - 6$$
$$6 < 4x$$
$$1.5 < x \quad or \quad \boldsymbol{x > 1.5}$$

c) **Hence**, write down the range of values of x which satisfy: $x + 4 < -x < 3x - 6$

$$x < -2 \quad and \quad x > 1.5 \quad \Rightarrow \quad \boldsymbol{No\ solutions\ for\ x}$$

Consider the graphs of the straight lines:

$y = -x$ $y = x + 4$ $y = 3x - 6$

There are three values of x where the lines cross one another: -2, 1.5 and 5.

Therefore there are four separate regions. In the first, the inequality from question 2 holds. In the second region, question 3 holds. In the third region, question 1 holds and in the fourth question 4 holds. Question 5 would only hold true if the line $y = x + 4$ was below both of the other two lines, and this is never the case.