How to factorise any quadratic starting with $x^2$

Notice that $(x + p)(x + q) = x^2 + px + qx + pq = x^2 + (p + q)x + pq$

This means that the numbers that will go in the brackets must:
• Multiply to make the constant term (number at the end)
• Add to make the $x$ coefficient (number in front of $x$)

Eg: \[x^2 + 9x + 20 = (x \ldots)(x \ldots)\]

I need two numbers which multiply to make 20 and add to make 9:
\[4 \times 5 = 20 \text{ and } 4 + 5 = 9\]

So ... \[x^2 + 9x + 20 = (x + 4)(x + 5)\]

What about negatives?
The rules above still work, but it is made easier by thinking about the different possibilities and what it means about the numbers. You can tell what sign the two numbers need to be just by looking at the sign of their product and their sum:

Complete these statements:
If the product of $p$ and $q$ is positive, and the sum of $p$ and $q$ is positive, then ...

If the product of $p$ and $q$ is positive, and the sum of $p$ and $q$ is negative, then ...

If the product of $p$ and $q$ is negative, and the sum of $p$ and $q$ is positive, then ...

If the product of $p$ and $q$ is negative, and the sum of $p$ and $q$ is negative, then ...

Factorise the following quadratics:

\[
\begin{array}{ccc}
x^2 + 8x + 15 & x^2 + 13x + 40 & x^2 + 1000x + 999 \\
x^2 - 6x + 8 & x^2 - 6x - 16 & x^2 + 12x - 45 \\
x^2 - 17x - 60 & 6x^2 - 42x - 360 & -x^3 - 15x^2 + 100x \\
\end{array}
\]

Hint: for the last two, remember that before you worry about double brackets, you can always use simple factoring techniques such as: $6x + 12 = 6(x + 2)$ and $-x^4 + x^2 = -x^2(x^2 - 1)$
How to factorise any quadratic starting with \( x^2 \) SOLUTIONS

Notice that \((x + p)(x + q) = x^2 + px + qx + pq = x^2 + (p + q)x + pq\)

This means that the numbers that will go in the brackets must:
- **Multiply** to make the constant term (number at the end)
- **Add** to make the \( x \) coefficient (number in front of \( x \))

Eg:

\[
x^2 + 9x + 20 = (x + \cdots)(x + \cdots)
\]

*I need two numbers which multiply to make 20 and add to make 9:*

\[4 \times 5 = 20 \quad \text{and} \quad 4 + 5 = 9\]

\[So \ldots \quad x^2 + 9x + 20 = (x + 4)(x + 5)\]

**What about negatives?**
The rules above still work, but it is made easier by thinking about the different possibilities and what it means about the numbers. You can tell what sign the two numbers need to be just by looking at the sign of their **product** and their **sum**:

*Complete these statements:*

If the **product** of \( p \) and \( q \) is positive, and the **sum** of \( p \) and \( q \) is positive, then ...

\[ p \text{ and } q \text{ must both be positive} \]

If the **product** of \( p \) and \( q \) is positive, and the **sum** of \( p \) and \( q \) is negative, then ...

\[ p \text{ and } q \text{ must both be negative} \]

If the **product** of \( p \) and \( q \) is negative, and the **sum** of \( p \) and \( q \) is positive, then ...

\[ \text{One will be positive, one negative (the larger will be positive)} \]

If the **product** of \( p \) and \( q \) is negative, and the **sum** of \( p \) and \( q \) is negative, then ...

\[ \text{One will be positive, one negative (the larger will be negative)} \]

**Factorise the following quadratics:**

<table>
<thead>
<tr>
<th>Quadratic</th>
<th>Factored Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 + 8x + 15 )</td>
<td>( = (x + 3)(x + 5) )</td>
</tr>
<tr>
<td>( x^2 - 6x + 8 )</td>
<td>( = (x - 2)(x - 4) )</td>
</tr>
<tr>
<td>( x^2 - 17x - 60 )</td>
<td>( = (x - 20)(x + 3) )</td>
</tr>
<tr>
<td>( x^2 + 13x + 40 )</td>
<td>( = (x + 5)(x + 8) )</td>
</tr>
<tr>
<td>( x^2 - 6x - 16 )</td>
<td>( = (x - 8)(x + 2) )</td>
</tr>
<tr>
<td>( 6x^2 - 42x - 360 )</td>
<td>( = 6(x^2 - 7x - 60) )</td>
</tr>
<tr>
<td>( x^2 + 1000x + 999 )</td>
<td>( = (x + 1)(x + 999) )</td>
</tr>
<tr>
<td>( x^2 + 12x - 45 )</td>
<td>( = (x - 5)(x + 9) )</td>
</tr>
<tr>
<td>( 6x^2 - 15x^2 + 100x )</td>
<td>( = -x(x^2 + 15x - 100) )</td>
</tr>
</tbody>
</table>

**Hint:** for the last two, remember that before you worry about double brackets, you can always use simple factoring techniques such as: \( 6x + 12 = 6(x + 2) \) and \( -x^4 + x^2 = -x^2(x^2 - 1) \)