

## Further Pure 2 Key Skills Checklist

*How confident are you with each topic?    $\surd$  confident   – not very sure    $\times$  very unsure*

<b>Chapter 1: Complex numbers</b>	
Recall and use the concept of a complex number in rectangular form ( $a + bi$ ).	
Know what is meant by the modulus of a complex number, and calculate it using Pythagoras' theorem.	
Know what is meant by the argument of a complex number, and calculate it using trigonometry.	
Be familiar with the polar form of a complex number, and convert between rectangular and polar form.	
Add or subtract complex numbers in rectangular form.	
Multiply complex numbers in rectangular form.	
Use the idea of a complex conjugate to divide complex numbers in rectangular form.	
Recall the results involving the modulus and argument of the product of two complex numbers.	
Recall the results involving the modulus and argument of the quotient of two complex numbers (division of two complex numbers).	
Multiply or divide complex numbers in polar form (aka modulus-argument form).	
Equate real parts and equate imaginary parts separately to solve equations where the unknown is a complex number.	
Recognise the geometric interpretation of $ z_2 - z_1 $ and $\arg(z_2 - z_1)$ in terms of the modulus and argument of the number $z_2 - z_1$ .	
Use complex numbers to describe the locus of a circle, centre $O$ , radius $k$ , on the Argand diagram.	
Use complex numbers to describe the locus of a circle, centre $z_1$ , radius $k$ , on the Argand diagram.	
Use complex numbers to describe the perpendicular bisector of a line segment joining the points $z_1$ and $z_2$ on the Argand diagram.	
Use complex numbers to describe the locus of a half line through $O$ , with angle to the positive $x$ direction of size $\alpha$ .	
Use complex numbers to describe the locus of a half line through $z_1$ , with angle to the positive $x$ direction of size $\alpha$ .	
Combine loci, including those involving inequalities, to describe a variety of conditions.	

<b>Chapter 2: Roots of polynomial equations</b>	
Solve any quadratic equation, giving solutions as complex numbers where necessary.	
Find the roots of a cubic equation by trial and error (for the real root) and solving the remaining quadratic, possibly to form complex roots.	
Recall and use the results for $\Sigma\alpha$ , $\Sigma\alpha\beta$ and $\alpha\beta\gamma$ (the sum of the roots, the sum of the product of pairs of the roots and the product of all three roots) to generate or modify cubic equations.	
Recall the result linking $\Sigma\alpha^2$ to $\Sigma\alpha$ and $\Sigma\alpha\beta$ .	
Extend the concepts above to any polynomial, and use them to find roots.	

### Chapter 3: Summation of finite series

Rearrange an expression to the form $f(x + 1) - f(x)$ .	
Apply the method of differences to a summation in order to determine the sum.	
Understand the concept of proof by induction.	
Apply proof by induction, including use of the appropriate statements, to prove summation results.	
Extend use of proof by induction to other areas of mathematics such as De Moivre's theorem.	

### Chapter 4: De Moivre's theorem and its applications

Reproduce De Moivre's theorem, and apply to simple situations.	
Prove De Moivre's theorem using proof by induction, and understand the proofs which extend its use to negative and fractional powers.	
Apply De Moivre's theorem to the problem of raising complex numbers to a power.	
Use De Moivre's theorem to generate trigonometrical identities for multiple angles in terms of powers of <i>sine</i> and <i>cosine</i> .	
Apply the methods above to form expressions for $\tan(n\theta)$ for given values of $n$ .	
Recall and use results involving $z + \frac{1}{z}$ and $z - \frac{1}{z}$ .	
Recall and use results involving $z^n + \frac{1}{z^n}$ and $z^n - \frac{1}{z^n}$ .	
Use De Moivre's theorem to generate trigonometrical identities for powers of <i>sine</i> and <i>cosine</i> in terms of multiple angles.	
Relate results from De Moivre's theorem to roots of polynomials.	
Use the exponential form of a complex number, and be familiar with Euler's Identity.	
Recall and use formulae for $\sin \theta$ and $\cos \theta$ involving the exponential form of a complex number.	
Use the properties of the cube roots of unity to solve problems.	
Extend ideas to $n^{\text{th}}$ roots of unity, including use of the Argand diagram.	
Find roots of numbers other than unity using the exponential form.	

### Chapter 5: Inverse trigonometrical functions

Plot the graphs of $y = \sin^{-1} x$ , $y = \cos^{-1} x$ and $y = \tan^{-1} x$ , noting the limited domain used to describe them as valid functions.	
Use implicit differentiation to derive the differential of the inverse trig functions.	
Incorporate product, quotient and chain rule to differentiation involving inverse trig.	
Prove the results (given in the formula book) for the integration of expressions such as $\frac{1}{a^2-x^2}$ and $\frac{1}{\sqrt{a^2-x^2}}$ .	
Use appropriate substitution along with the standard results described above to solve more complex integration problems.	
Incorporate substitution and the $\int \frac{f'(x)}{f(x)}$ rule in order to solve integration problems.	

### Chapter 6: Hyperbolic functions

Know the definitions for the hyperbolic functions $\sinh x$ and $\cosh x$ in terms of $e^x$ .	
Derive the four other hyperbolic functions from these two.	
Use the exponential form to calculate exact values for hyperbolic functions of values.	
Be familiar with, and be able to sketch, the graphs of all the hyperbolic functions.	
Use the exponential form to prove hyperbolic identities.	
Use established identities to rearrange and simplify expressions involving hyperbolic functions.	
Recall and use Osborne's rule (note: not to be used in proofs).	
Differentiate hyperbolic functions by converting to exponential form.	
Use established results to differentiate expressions involving hyperbolic functions.	
Integrate hyperbolic functions by converting to exponential form, or directly using established results and methods of integration.	
Use the exponential form to generate definitions of inverse hyperbolic functions involving logarithms.	
Use implicit differentiation to derive the differential of inverse hyperbolic functions.	
Recognise and use the standard results (from the formula book) for integrals of the functions $\frac{1}{\sqrt{a^2+x^2}}$ , $\frac{1}{\sqrt{x^2-a^2}}$ and $\frac{1}{\sqrt{a^2-x^2}}$ .	
Solve equations involving hyperbolic functions by converting to exponential form and writing as a quadratic in $e^x$ .	

### Chapter 7: Arc length and area of surface of revolution

Understand and apply the standard formula (in the formula book) for arc length.	
Apply the parametric formula (in the formula book) for arc length where necessary.	
Understand and apply the standard formula (in the formula book) for surface area of revolution.	
Apply the parametric formula (in the formula book) for surface area of revolution where necessary.	