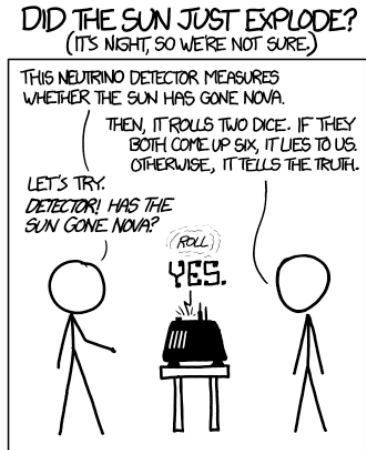
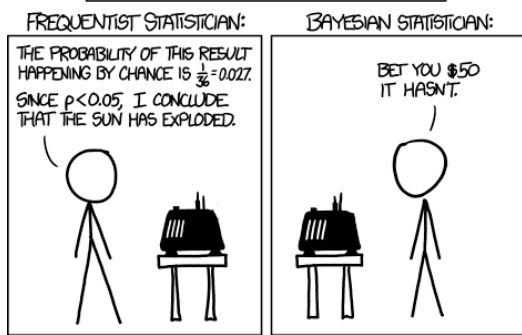


Exploding Stars: Conditional Probability from a Tree Diagram



Some probabilities can be calculated directly (theoretical). Eg the chance of rolling two sixes with a pair of dice is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

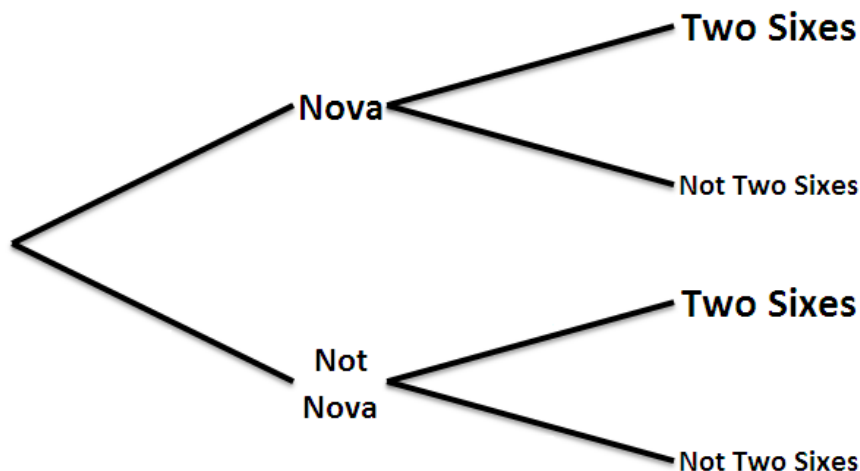
Some are estimated from observations (experimental). Eg the chance of rain on a random day in the UK is around $\frac{133}{365}$ based on previous years.



Some probabilities cannot be found either theoretically or experimentally. The most famous is: “What’s the chance of the sun rising tomorrow?” We can’t calculate it directly, and (because we only have one sun and it hasn’t exploded yet) we can’t estimate it experimentally.

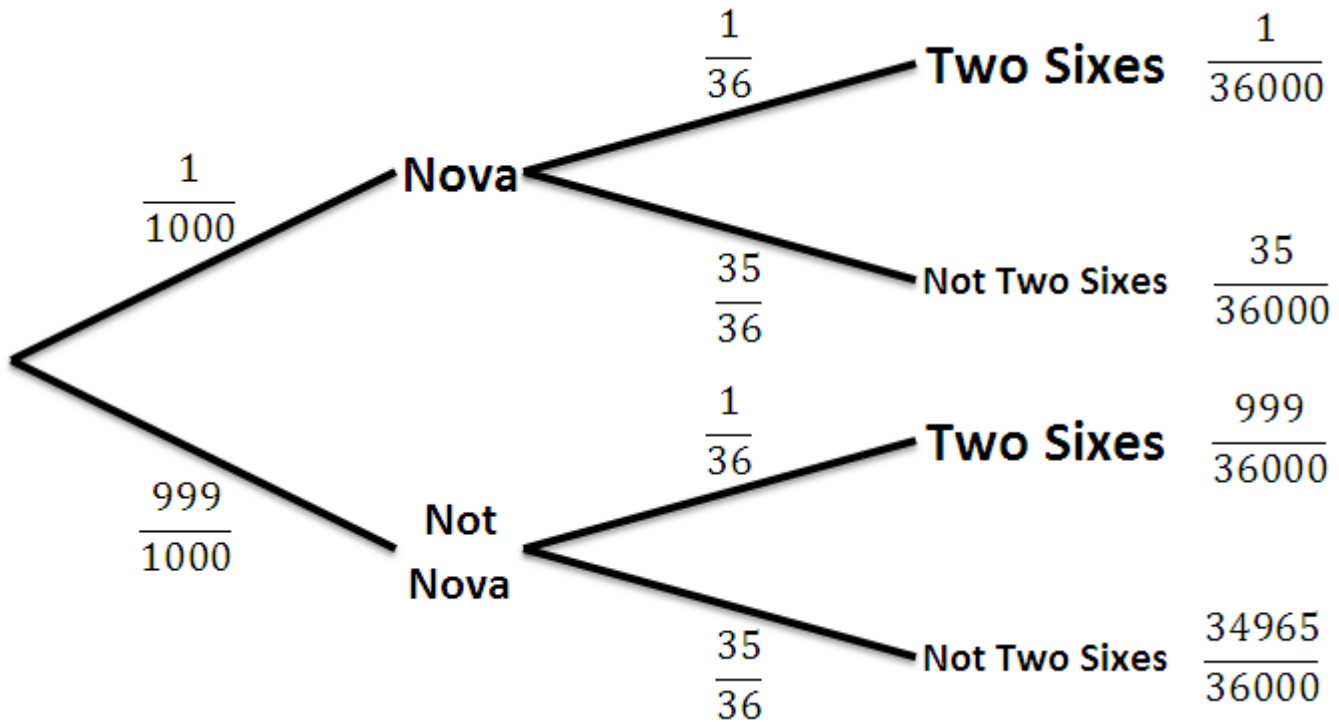
Although mathematicians disagree about the best way to answer questions like this, it makes the most practical sense to assume *some* probability when dealing with an unknowable problem like the sun going nova.

By putting probabilities on the tree diagram below, assuming a $\frac{1}{1000}$ chance that the sun has gone nova overnight, decide whether you think the machine is telling the truth (and we’re doomed) or lying (and we’re actually fine).



Exploding Stars SOLUTIONS

The tree diagram should look like this:



The most likely outcome is that the sun has not gone nova, and the machine didn't roll two sixes. However, given that the machine told us we are all doomed, two possibilities remain:

1. The sun has exploded, the machine didn't roll 2 sixes (so told the truth): $\frac{35}{36000}$
2. The sun has not exploded, the machine did roll 2 sixes (and lied to us): $\frac{999}{36000}$

Since the second is much more likely, our prior hunch that the sun is unlikely to explode overnight allows us to guess that the machine was lying.

The chance of the machine having lied, given that it told us the sun had exploded:

$$\frac{999}{35 + 999} = \frac{999}{1034}$$

If we reduce our generous estimate for the probability of the sun exploding even further, we get an even more extreme result.

Ext: Can you think of another reason the second statistician should bet against his friend?