

Euclid's Algorithm for finding the highest common factor of two positive integers:

<pre>def hcf(a,b): i=0 while a*b>0: if a>b: a,b=b,a b=b-a i=i+1 print "a="+str(a)+",b="+str(b)+", i= "+str(i) print "HCF="+str(a)</pre>	<p>The function is defined, taking two input values: a and b.</p> <p>The variable i is introduced to count the number of iterations required.</p> <p>A 'while' loop repeats till one or other of a & b is 0</p> <p>The 'if' statement checks which is the largest...</p> <p>...and redefines (swaps a and b) if required to ensure b>a</p> <p>b is now replaced by the difference b-a</p> <p>This counts as one more iteration, so increase i</p> <p>Print information on the current values of a, b and i (while loop continues as long as a*b>0)</p> <p>Print the highest common factor (the current value of a)</p>
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The modulus function allows you to find the remainder when one positive integer is divided by another. In Python, this function can be called using a%b ('a modulo b', or 'the remainder when a is divided by b').

Eg: 38%5 returns 3 since $\frac{38}{5} = 7\frac{3}{5}$.

Can you incorporate this function to come up with a more efficient algorithm to find the highest common factor?

Hint: consider what the algorithm above does when a=100 and b=15, and how 100%15 could provide a short cut.

Solution:

Improved algorithm	<pre>def hcf2(a,b): while a*b>0: if a>b: a,b=b,a b=b%a print "HCF="+str(a)</pre>	(simplified form)	<pre>def hcf(a,b): while a*b>0: if a>b: a,b=b,a b=b-a print "HCF="+str(a)</pre>
Replaces the largest of a and b with the remainder when the largest is divided by the smallest.		Replaces the largest of a and b with the difference between a and b each time.	

Note: this improved algorithm is capable of finding the HCF of any two numbers in at most 5 times the number of digits of the smallest of the two numbers.