Enlargement

Once you have completed this booklet you should be able to:

- Understand how enlargement changes the lengths of the lines in a shape
- Enlarge shapes by scale factors such as 2 or 3
- Find the point of enlargement (vanishing point) for a given enlargement using ray lines
- Use a point of enlargement to enlarge a shape by positive whole number scale factors
- Investigate how enlarging lengths affects area
- Investigate the effect of fractional or negative scale factors

Section A: Interpreting enlargements

The scale factor of an enlargement is how much bigger the new shape’s lengths are. This is what you multiply by to increase the size, not what you add (to keep the shape the same).

1. The larger rectangle is an enlargement of the smaller one:

   The width of the small rectangle is: ___ cm
   The width of the large rectangle is: ___ cm
   The large rectangle is ___ times wider.

   The height of the small rectangle is: ___ cm
   The height of the large rectangle is: ___ cm
   The large rectangle is ___ times higher.

   The large rectangle is an enlargement of the small rectangle by scale factor ____.

2. The larger rectangle is an enlargement of the smaller one:

   The width of the small rectangle is: ___ cm
   The width of the large rectangle is: ___ cm
   The large rectangle is ___ times wider.

   The height of the small rectangle is: ___ cm
   The height of the large rectangle is: ___ cm
   The large rectangle is ___ times higher.

   The large rectangle is an enlargement of the small rectangle by scale factor ____.

3. The larger parallelogram is an enlargement of the smaller one:

   The width of the small parallelogram is: ___ cm
   The width of the large parallelogram is: ___ cm
   The large parallelogram is ___ times wider.

   The height of the small parallelogram is: ___ cm
   The height of the large parallelogram is: ___ cm
   The large parallelogram is ___ times higher.

   The large parallelogram is an enlargement of the small parallelogram by scale factor ____.
Section B: Enlarging simple shapes

Don’t turn or flip the shape when you make it larger, but you can put the new shape anywhere.

1. Draw an enlargement of the square below. Make all the lengths 2 times longer.

The new shape is an enlargement from the original shape of scale factor ____.

2. Draw an enlargement of the square below. Make all the lengths 3 times longer.

This is an enlargement of scale factor ____.

3. Enlarge the rectangle by scale factor 2, then enlarge this new shape by scale factor 3.

The final shape is an enlargement from the original shape by scale factor ____.

4. Enlarge the trapezium below by scale factor 2:

Remember to double all lengths.

The easiest way is to start with the horizontal and vertical lines, and leave the slanting line until last.

5. a) Enlarge the L shape below by scale factor 2:

b) How many copies of the original shape are needed to completely fill the new (enlarged) shape?

It is possible to do this just with rotated copies of the original, but if you can’t see how, you’re allowed to cut them up!
Section C: Finding the point of enlargement

If you draw lines back from the enlarged shape to the original, the all meet at a point.

Example: The larger shape is an enlargement by scale factor 2 from the shape on the left.

Joining up the top-left corners of both shapes, the top-right corners, etc, and extending lines:

The point where all the lines meet is called the point of enlargement (or vanishing point).

1. a) Draw ray lines from each corner of the larger shape through the same corner of the original shape, and continue the lines as far as you can.

b) Label the point where all four ray lines cross with a ×. This is the point of enlargement.
2. The parallelogram below has been enlarged by scale factor 2.5. Find the point of enlargement using ray lines from corresponding corners.

3. a) The kite below has been enlarged. Find the point of enlargement. (mark it ×)

b) The bottom-right corner of the small kite is _____ cm away from the point of enlargement.

c) The bottom-right corner of the large kite is _____ cm away from the point of enlargement.

d) The scale factor of enlargement is ____.

4. The small triangle has been enlarged twice with different scale factors from the same point.

a) Find the point of enlargement which has been used for both enlargements (mark it ×).

b) The middle triangle is an enlargement from the smallest by scale factor: _____

c) The largest triangle is an enlargement from the smallest by scale factor: ____
Section D: Enlarging a shape from a point

By drawing ray lines from a point to the corners of your shape, then extending them further, you can create enlargements with a given vanishing point.

1. The larger trapezium is an enlargement of the smaller one.
   a) The bottom-left corner of the small trapezium is ____ cm away from the point of enlargement.
   b) The bottom-left corner of the large trapezium is ____ cm away from the point of enlargement.
   c) The scale factor is ____.

The enlargement shown below is the same as the one above.
   d) The top-left corner of the small trapezium is...
      _____ cm to the right, and
      _____ cm up
      ...from the point of enlargement.
   e) The top-left corner of the large trapezium is...
      _____ cm to the right, and
      _____ cm up
      ...from the point of enlargement.

f) On one of the diagrams above, draw in the additional ray lines as arrows to show how each point on the larger shape is 2 times further from the point of enlargement as the smaller shape.

2. a) On the diagram below, draw ray lines from the point of enlargement (marked ×) to each of the four corners of the rectangle.
   b) Extend each of your four lines so they are 2 times longer and mark the four end points with ×.
   c) Join up the end points of your lines to form a new rectangle.
   d) This new rectangle is an enlargement from the original rectangle by scale factor: ____.
3. Enlarge the following rectangles by scale factor 2 from the marked point of enlargement:

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4. a) Enlarge this parallelogram by scale factor 3 from the marked point of enlargement:

b) How many copies of the original parallelogram can fit in the enlarged parallelogram?

c) The area of the original parallelogram is $6cm^2$. What is the area of the enlarged one?
Section E: Negative and fractional scale factors

Multiplying a length by $\frac{1}{2}$ gives a new length half the size. Multiplying by a negative number changes the direction of the line, so a $-3$ scale factor gives a line 3 times longer, but backwards.

1. In each diagram below, the original (grey) shape has been enlarged from the point of enlargement to give the dotted red shape.

   Choose from the list below the correct scale factor to write beside each shape:
   \[ \text{Scale Factor 0.5} \quad \text{Scale Factor 0.1} \quad \text{Scale Factor 1.5} \quad \text{Scale Factor 0.25} \]

2. In each diagram below, the original (grey) shape has been enlarged from the point of enlargement to give the dotted blue shape.

   Choose from the list below the correct scale factor to write beside each shape:
   \[ \text{Scale Factor } -1 \quad \text{Scale Factor } -2 \quad \text{Scale Factor } -0.5 \quad \text{Scale Factor 0} \]
3. Each of the four diagrams below represent a different transformation, from these options:
- **Reflection** in line $L$.
- **Enlargement** by scale factor $-1$ from point $A$.
- **Rotation** of $180^\circ$ around point $A$.
- **Translation** by vector $\begin{bmatrix} -9 \\ 0 \end{bmatrix}$.

Write next to each diagram whether it is a **reflection**, **enlargement**, **rotation** or **translation**:

4. a) Enlarge the shape below by **scale factor 0.5** from the point of enlargement (marked $\times$):

b) What is the **area** of the original shape?  
c) What is the **area** of the new shape?

d) The **length scale factor** is 0.5 (this is what we use to describe the enlargement).  
   What is the **area scale factor**?
Enlargement SOLUTIONS

Once you have completed this booklet you should be able to:
- Understand how enlargement changes the lengths of the lines in a shape
- Enlarge shapes by scale factors such as 2 or 3
- Find the point of enlargement (vanishing point) for a given enlargement using ray lines
- Use a point of enlargement to enlarge a shape by positive whole number scale factors
- Investigate how enlarging lengths affects area
- Investigate the effect of fractional or negative scale factors

Section A: Interpreting enlargements SOLUTIONS

The scale factor of an enlargement is how much bigger the new shape’s lengths are.
This is what you multiply by to increase the size, not what you add (to keep the shape the same)

1. The larger rectangle is an enlargement of the smaller one:

   The width of the small rectangle is: 2 cm
   The width of the large rectangle is: 4 cm
   The large rectangle is 2 times wider.

   The height of the small rectangle is: 3 cm
   The height of the large rectangle is: 6 cm
   The large rectangle is 2 times higher.

   The large rectangle is an enlargement of the small rectangle by scale factor 2.

2. The larger rectangle is an enlargement of the smaller one:

   The width of the small rectangle is: 2 cm
   The width of the large rectangle is: 6 cm
   The large rectangle is 3 times wider.

   The height of the small rectangle is: 1 cm
   The height of the large rectangle is: 3 cm
   The large rectangle is 3 times higher.

   The large rectangle is an enlargement of the small rectangle by scale factor 3.

3. The larger parallelogram is an enlargement of the smaller one:

   The width of the small parallelogram is: 1 cm
   The width of the large parallelogram is: 5 cm
   The large parallelogram is 5 times wider.

   The height of the small parallelogram is: 2 cm
   The height of the large parallelogram is: 10 cm
   The large parallelogram is 5 times higher.

   The large parallelogram is an enlargement of the small parallelogram by scale factor 5.
Section B: Enlarging simple shapes SOLUTIONS

Don’t turn or flip the shape when you make it larger, but you can put the new shape anywhere.

1. Draw an enlargement of the square below. Make all the lengths 2 times longer.

The new shape is an enlargement from the original shape of scale factor 2.

2. Draw an enlargement of the square below. Make all the lengths 3 times longer.

This is an enlargement of scale factor 3.

3. Enlarge the rectangle by scale factor 2, then enlarge this new shape by scale factor 3.

The final shape is an enlargement from the original shape by scale factor 6.

4. Enlarge the trapezium below by scale factor 2:

Remember to double all lengths.

The easiest way is to start with the horizontal and vertical lines, and leave the slanting line until last.

5. a) Enlarge the L shape below by scale factor 2:

b) How many copies of the original shape are needed to completely fill the new (enlarged) shape?

4

It is possible to do this just with rotated copies of the original, but if you can’t see how, you’re allowed to cut them up!
Section C: Finding the point of enlargement **SOLUTIONS**
If you draw lines back from the enlarged shape to the original, the all meet at a point.

**Example:** The larger shape is an enlargement by scale factor 2 from the shape on the left.

Joining up the top-left corners of both shapes, the top-right corners, etc, and extending lines:

The point where all the lines meet is called the **point of enlargement** (or **vanishing point**).

1. a) Draw ray lines from each corner of the larger shape through the **same corner** of the original shape, and continue the lines as far as you can.

b) Label the point where all four ray lines cross with a ×. This is the **point of enlargement**.
2. The parallelogram below has been enlarged by scale factor 2.5.  
Find the point of enlargement using ray lines from corresponding corners.

3. a) The kite below has been enlarged.  
Find the **point of enlargement**. (mark it \( \times \))

   b) The bottom-right corner of the small kite is \( 1 \) cm away from the point of enlargement.

   c) The bottom-right corner of the large kite is \( 3 \) cm away from the point of enlargement.

   d) The **scale factor** of this enlargement is \( 3 \).

4. The small triangle has been enlarged **twice** with different scale factors from the same point.

   a) Find the **point of enlargement** which has been used for both enlargements (mark it \( \times \)).

   b) The **middle** triangle is an enlargement from the **smallest** by scale factor: \( 2 \)

   c) The **largest** triangle is an enlargement from the **smallest** by scale factor: \( 3 \).
Section D: Enlarging a shape from a point **SOLUTIONS**

**By drawing ray lines from a point to the corners of your shape, then extending them further, you can create enlargements with a given vanishing point.**

1. The larger trapezium is an enlargement of the smaller one.
   a) The bottom-left corner of the small trapezium is 3 cm away from the point of enlargement.
   b) The bottom-left corner of the large trapezium is 6 cm away from the point of enlargement.
   c) The scale factor is 2.

   The enlargement shown below is the same as the one above.
   d) The top-left corner of the small trapezium is...
      - 3 cm to the right, and
      - 2 cm up
      ...from the point of enlargement.
   e) The top-left corner of the large trapezium is...
      - 6 cm to the right, and
      - 4 cm up
      ...from the point of enlargement.
   f) On one of the diagrams above, draw in the additional ray lines as arrows to show how each point on the larger shape is **2 times further** from the point of enlargement as the smaller shape.

2. a) On the diagram below, draw ray lines from the point of enlargement (marked ×) to each of the four corners of the rectangle.

   b) Extend each of your four lines so they are **2 times longer** and mark the four end points with ×.

   c) Join up the end points of your lines to form a new rectangle.

   d) This new rectangle is an enlargement from the original rectangle by **scale factor: 2**.
3. Enlarge the following rectangles by **scale factor 2** from the marked point of enlargement:

a)  

b)  

c)  

d)  

4. a) Enlarge this parallelogram by **scale factor 3** from the marked point of enlargement:

b) How many copies of the original parallelogram can fit in the enlarged parallelogram?  

9  

c) The area of the original parallelogram is $6cm^2$. What is the area of the enlarged one?  

$6 \times 9 = 54cm^2$
Section E: Negative and fractional scale factors

Multiplying a length by $\frac{1}{2}$ gives a new length half the size. Multiplying by a negative number changes the direction of the line, so a $-3$ scale factor gives a line 3 times longer, but backwards.

1. In each diagram below, the original (grey) shape has been enlarged from the point of enlargement to give the dotted red shape. Choose from the list below the correct scale factor to write beside each shape:

   - Scale Factor 0.5
   - Scale Factor 0.1
   - Scale Factor 1.5
   - Scale Factor 0.25

![Diagram 1](image1)

2. In each diagram below, the original (grey) shape has been enlarged from the point of enlargement to give the dotted blue shape. Choose from the list below the correct scale factor to write beside each shape:

   - Scale Factor $-1$
   - Scale Factor $-2$
   - Scale Factor 0.5
   - Scale Factor 0

![Diagram 2](image2)
3. Each of the four diagrams below represent a different transformation, from these options:
   - **Reflection** in line \( L \).
   - **Enlargement** by scale factor \(-1\) from point \( A \).
   - **Rotation** of \(180^\circ\) around point \( A \).
   - **Translation** by vector \([-9, 0]\).

Write next to each diagram whether it is a reflection, enlargement, rotation or translation:

4. a) Enlarge the shape below by **scale factor 0.5** from the point of enlargement (marked \( \times \)):

   b) What is the **area** of the original shape?  
      - \(10cm^2\)

   c) What is the **area** of the new shape?  
      - \(2.5cm^2\)

   d) The **length scale factor** is 0.5 (this is what we use to describe the enlargement).
   
   What is the **area scale factor**?  
   - \(0.25\)