Compound Interest

Interest on savings is usually paid annually.

£100 invested at 6.24% yields: $100 \times 1.0624 = £106$.

A bank offers $\frac{1}{12}$ the rate (0.52%), paid monthly.
Because interest is compounded, this will earn you slightly more:

$$100 \times 1.0052^{12} \approx £106.42$$

1. How much would you end up with after a year if you were paid $\frac{1}{52}$ the rate (0.12%), compounded weekly?

2. How much would you end up with after a year if $\frac{1}{365}$ of the original annual rate were paid, compounded daily?

3. Form an expression for the **multiplier** for a single year when money is invested at an annual rate of p (where p represents the interest, written as a decimal. Eg 6%: $p = 0.06$).

4. The interest rate is now divided into n equal parts, and is payable, compounded, n times per year. Modify your expression accordingly.

5. Substitute the values $p = 0.0624$ and $n = 8760$ to determine the multiplier when an investment is made at a rate of 6.24%, shared between, and compounded, hourly.

6. Note that, substituting $y = \frac{n}{p}$ into your expression from question 4 yields: $\left(\left(1 + \frac{1}{y}\right)^y\right)^p$.
If there is a limit to how much you can get from your investment, $\left(1 + \frac{1}{y}\right)^y$ must be limited.
Investigate what happens as $y \to \infty$ (that is, as y gets larger and larger).

Compound Interest SOLUTIONS

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1. How much would you end up with after a year if you were paid $\frac{1}{52}$ the rate (0.12%), compounded weekly?

$$100 \times 1.0012^{52} \approx £106.43$$

2. How much would you end up with after a year if $\frac{1}{365}$ of the original annual rate were paid, compounded daily?

$$100 \times \left(1 + \frac{0.0624}{365}\right)^{365} \approx £106.44$$

3. Form an expression for the **multiplier** for a single year when money is invested at an annual rate of p (where p represents the interest, written as a decimal. Eg 6%: $p = 0.06$).

$$(1 + p)$$

4. The interest rate is now divided into n equal parts, and is payable, compounded, n times per year. Modify your expression accordingly.

$$\left(1 + \frac{p}{n}\right)^n$$

5. Substitute the values $p = 0.0624$ and $n = 8760$ to determine the multiplier when an investment is made at a rate of 6.24%, shared between, and compounded, hourly.

$$\left(1 + \frac{0.0624}{8760}\right)^{8760} \approx 1.06439$$

6. Note that, substituting $y = \frac{n}{p}$ into your expression from question 4 yields: $\left(\left(1 + \frac{1}{y}\right)^y\right)^p$.

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Investigate what happens as $y \to \infty$ (that is, as y gets larger and larger).

$$\left(1 + \frac{1}{1000}\right)^{1000} \approx 2.7169 \quad \left(1 + \frac{1}{1000000}\right)^{1000000} \approx 2.7183 \quad \left(1 + \frac{1}{y}\right)^y \to e \ as \ y \to \infty$$