

**Compound Interest**

Interest on savings is usually paid annually.

£100 invested at 6.24% yields: $100 \times 1.0624 = £106$.

A bank offers $\frac{1}{12}$ the rate (0.52%), paid monthly. Because interest is compounded, this will earn you slightly more:

$100 \times 1.0052^{12} \approx £106.42$

1. How much would you end up with after a year if you were paid $\frac{1}{52}$ the rate (0.12%), compounded weekly?

2. How much would you end up with after a year if $\frac{1}{365}$ of the original annual rate were paid, compounded daily?

3. Form an expression for the **multiplier** for a single year when money is invested at an annual rate of $p$ (where $p$ represents the interest, written as a decimal. Eg 6%: $p = 0.06$).

4. The interest rate is now divided into $n$ equal parts, and is payable, compounded, $n$ times per year. Modify your expression accordingly.

5. Substitute the values $p = 0.0624$ and $n = 8760$ to determine the multiplier when an investment is made at a rate of 6.24%, shared between, and compounded, hourly.

6. Note that, substituting $y = \frac{n}{p}$ into your expression from question 4 yields: $\left(\left(1 + \frac{1}{y}\right)^y\right)^p$.

If there is a limit to how much you can get from your investment, $\left(1 + \frac{1}{y}\right)^y$ must be limited. Investigate what happens as $y \to \infty$ (that is, as $y$ gets larger and larger).
Compound Interest SOLUTIONS

Interest on savings is usually paid annually.

£100 invested at 6.24% yields: \( 100 \times 1.0624 = £106.24 \).

A bank offers \( \frac{1}{12} \) the rate (0.52%), paid monthly. Because interest is compounded, this will earn you slightly more:

\[
100 \times 1.0052^{12} \approx £106.42
\]

1. How much would you end up with after a year if you were paid \( \frac{1}{52} \) the rate (0.12%), compounded weekly?

\[
100 \times 1.0012^{52} \approx £106.43
\]

2. How much would you end up with after a year if \( \frac{1}{365} \) of the original annual rate were paid, compounded daily?

\[
100 \times \left(1 + \frac{0.0624}{365}\right)^{365} \approx £106.44
\]

3. Form an expression for the multiplier for a single year when money is invested at an annual rate of \( p \) (where \( p \) represents the interest, written as a decimal. Eg 6%: \( p = 0.06 \)).

\[
(1 + p)
\]

4. The interest rate is now divided into \( n \) equal parts, and is payable, compounded, \( n \) times per year. Modify your expression accordingly.

\[
\left(1 + \frac{p}{n}\right)^n
\]

5. Substitute the values \( p = 0.0624 \) and \( n = 8760 \) to determine the multiplier when an investment is made at a rate of 6.24%, shared between, and compounded, hourly.

\[
\left(1 + \frac{0.0624}{8760}\right)^{8760} \approx 1.06439
\]

6. Note that, substituting \( y = \frac{n}{p} \) into your expression from question 4 yields: \( \left(1 + \frac{1}{y}\right)^y \).

If there is a limit to how much you can get from your investment, \( \left(1 + \frac{1}{y}\right)^y \) must be limited. Investigate what happens as \( y \to \infty \) (that is, as \( y \) gets larger and larger).

\[
\left(1 + \frac{1}{1000}\right)^{1000} \approx 2.7169 \quad \left(1 + \frac{1}{1000000}\right)^{1000000} \approx 2.7183 \quad \left(1 + \frac{1}{y}\right)^y \to e \text{ as } y \to \infty
\]