Circle Theorems

Circle Facts:

1. Any triangle with two points on the edge of a circle and one in the middle will be isosceles.
2. The tangent to a circle is perpendicular to the radius at the point of contact.
3. The triangle produced by two tangents to a circle and the chord between them is isosceles.
4. If a radius bisects a chord, it does so at right angles, and if it cuts it at right angles it bisects it.

Circle Theorems:

1. Double Angle: The angle made at the centre of a circle is twice the angle made at the edge.
2. Semicircle: The angle in a semicircle is a right angle.
3. Segment Angles: Angles in the same segment are equal.
4. Cyclic Quadrilateral: Opposite angles of a cyclic quadrilateral add up to $180^\circ$.
5. Alternate segment: The angle between a chord and the tangent at the point of contact is equal to the angle in the alternate segment.
Circle Theorems Questions

1.

In the diagram, A, B, C and D are points on the circumference of a circle, centre O.
Angle BAD = 70°.
Angle BOD = \(x\)°.
Angle BCD = \(y\)°.

(a) (i) Work out the value of \(x\).

\[ x = \text{.................................} \]

(ii) Give a reason for your answer.

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(b) (i) Work out the value of \(y\).

\[ y = \text{.................................} \]

(ii) Give a reason for your answer.

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(Total 4 marks)
2.

Diagram NOT accurately drawn

A, B and C are points on the circumference of a circle, centre O. 
AC is a diameter of the circle.

(a) (i) Write down the size of angle ABC.

..........................°

(ii) Give a reason for your answer.
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(b) (i) Work out the size of angle DEF.

..........................°

(ii) Give a reason for your answer.
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Diagram NOT accurately drawn

D, E and F are points on the circumference of a circle, centre O. 
Angle DOF = 130°.

(Total 4 marks)
The diagram shows a circle centre $O$. $A$, $B$ and $C$ are points on the circumference.

$DCO$ is a straight line.
$DA$ is a tangent to the circle.

Angle $ADO = 36^\circ$

(a) Work out the size of angle $AOD$.

(b) (i) Work out the size of angle $ABC$.

(ii) Give a reason for your answer.

(Total 5 marks)
4.

Diagram NOT accurately drawn

$R$ and $S$ are two points on a circle, centre $O$.
$TS$ is a tangent to the circle.
Angle $RST = x$.

Prove that angle $ROS = 2x$.
You must give reasons for each stage of your working.
B and C are points on a circle, centre O.
AB and AC are tangents to the circle.
Angle $BOC = 130^\circ$.

Work out the size of angle $BAO$. 

Diagram NOT accurately drawn

...............°

(Total 3 marks)
$B$ and $C$ are two points on a circle, centre $O$.

Angle $OBC = 15^\circ$.

$AB$ and $AC$ are tangents to the circle.

(a) Calculate the size of the angle marked $x^\circ$.

(b) Give reasons for your answer.

\[
\begin{align*}
\text{Diagram NOT accurately drawn}\end{align*}
\]

\[x^\circ\] (2)

(Total 4 marks)
A and B are points on a circle, centre O, radius 3 cm.

PA and PB are tangents to the circle.

PA = 5 cm.

(a) Write down the size of the angle OBP.

..................°

(b) (i) Write down the length of PB.

................. cm

(ii) Give a reason for your answer.

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(Total 3 marks)
8. The diagram shows a circle, centre $O$.
$A$, $S$, $B$ and $T$ are points on the circumference of the circle.

$PT$ and $PS$ are tangents to the circle.
$AB$ is parallel to $TP$.

Angle $SPT = 44^\circ$.

Work out the size of angle $SOB$.  

..............................°  

(Total 4 marks)
$A$ and $B$ are points on the circumference of a circle, centre $O$.  
$PA$ and $PB$ are tangents to the circle.  
Angle $APB$ is $86^\circ$.  

Work out the size of the angle marked $x$.  

Diagram NOT accurately drawn  

\[
\begin{align*}
\text{Angle } APB &= 86^\circ \\
\text{Work out the size of the angle marked } x.
\end{align*}
\]
In the diagram, $A$, $B$ and $C$ are points on the circumference of a circle, centre $O$.

Angle $ABC = 85^\circ$.

(i) Work out the size of the angle marked $x^\circ$.

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(ii) Give a reason for your answer.

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(Total 2 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Circle Theorem Hints</th>
<th>Circle Theorem Answers</th>
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<tbody>
<tr>
<td>Question 1</td>
<td>You will need circle theorems 1 and 4.</td>
<td>$x = 140^\circ$ because the angle made at the centre of a circle is twice the angle made at the edge. $y = 110^\circ$ because opposite angles of a cyclic quadrilateral add up to $180^\circ$.</td>
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<tr>
<td>Question 2</td>
<td>You will need circle theorems 2 and 1.</td>
<td>$ABC = 90^\circ$ because the angle in a semicircle is a right angle. $DEF = 65^\circ$ because the angle made at the centre of a circle is twice the angle made at the edge.</td>
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<tr>
<td>Question 3</td>
<td>You will need circle fact 2 and circle theorem 1.</td>
<td>$AOD = 54^\circ$ because $DAO = 90^\circ$. $ABC = 27^\circ$ because the angle made at the centre of a circle is twice the angle made at the edge.</td>
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<tr>
<td>Question 4</td>
<td>You will need circle facts 2 and 1.</td>
<td>$TSO = 90^\circ$ because the tangent to a circle is perpendicular to the radius at the point of contact. Therefore angle $OSR = 90 - x$. Triangle $SOR$ is isosceles since $SO$ and $OR$ are both the radius of the circle, therefore $ORS = 90 - x$ and $ROS = 180 - (180 - 2x) = 2x$.</td>
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<tr>
<td>Question 5</td>
<td>You will need circle facts 2 and 3.</td>
<td>Angle $ABO = 90^\circ$ because $AB$ is a tangent and $BO$ is a radius. Similarly, $ACO = 90^\circ$. The angles in the quadrilateral $ABOC$ must add up to $180^\circ$, so since $BOC = 130^\circ$, $BAC = 50^\circ$. Triangle $ABC$ is isosceles, and triangle $BOC$ is isosceles. Since they are both symmetrical about $AO$, the line $AO$ bisects the angle $BAO$. Therefore $BAO = 25^\circ$.</td>
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<tr>
<td>Question 6</td>
<td>You will need circle facts 2 and 3.</td>
<td>Triangle $BOC$ is isosceles since $BO$ and $CO$ are both the radius of the circle. Therefore $OCB = 15^\circ$ and $BOC = 180 - 2(15) = 150^\circ$. Since $ABO = 90^\circ$ and $ACO = 90^\circ$ (as $AB$ and $AC$ are tangents and $BO$ and $CO$ radii), the fourth angle in the quadrilateral $ABOC$ - the angle $x$ - must add to $150^\circ$, $90^\circ$ and $90^\circ$ to make $360^\circ$, therefore $x = 30^\circ$.</td>
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<tr>
<td>Question 7</td>
<td>You will need circle facts 2 and 3</td>
<td>$OBP = 90^\circ$ because $PB$ is a tangent to the circle at $B$, and $BO$ is a radius. $PB = 5cm$ because triangle $PBA$ is isosceles, therefore $PB = PA$.</td>
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<tr>
<td>Question 8</td>
<td>You will need circle fact 2.</td>
<td>Angle $PTO = 90^\circ$, angle $PSO = 90^\circ$ and angle $TPS = 44^\circ$. The fourth angle in the quadrilateral $TPSO$ must therefore be $TOS = 136^\circ$. Angle $TOB = 90^\circ$ because line $BA$ is parallel to line $PT$ and interior angles on parallel lines add up to $180^\circ$. Therefore $SOB = 136 - 90 = 46^\circ$.</td>
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<tr>
<td>Question 9</td>
<td>You will need circle facts 2 and 3.</td>
<td>Since $OAP$ and $OBP$ are $90^\circ$ ($AP$ and $BP$ are tangents to the circle, and $OA$ and $OB$ are radii), $AOB = 180 - 86 = 94^\circ$. Triangle $AOB$ is isosceles, so $x = \frac{180 - 94}{2} = 43^\circ$.</td>
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<tr>
<td>Question 10</td>
<td>You will need circle theorem 1.</td>
<td>$x = 170^\circ$ because the angle made at the centre of a circle is twice the angle made at the edge.</td>
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