1. Use Simpson’s rule, with seven ordinates (six strips), to calculate an estimate for
\[ \int_{0}^{\pi/2} x^2 \cos x \, dx \]
Give your answer to four significant figures. [4 marks]

2. A curve has equation \( y = \frac{\ln(e-2x)}{2} \)
   a) Find \( \frac{dy}{dx} \). [2 marks]
   b) Find an equation of the normal to the curve \( y = \frac{\ln(e-2x)}{2} \) at the point on the curve where \( x = -e \). [4 marks]
   c) The curve \( y = \frac{\ln(e-2x)}{2} \) crosses the line \( y = x \) at \( x = \alpha \).
      i. Show that \( \alpha \) lies between 0 and 1. [2 marks]
      ii. Use the recurrence relation \( x_{n+1} = \frac{\ln(e-2x_n)}{2} \) with \( x_1 = 0 \) to find the values of \( x_2 \) and \( x_3 \) to 4 significant figures. [2 marks]

3. a) i. Differentiate \((3 - x^3)^{\frac{3}{2}}\) with respect to \( x \). [2 marks]
      ii. Given that \( y = e^x(3 - x^3)^{\frac{3}{2}} \), find the exact value of \( \frac{dy}{dx} \) when \( x = 0 \). [3 marks]
   b) A curve has equation \( y = \frac{x-1}{x^2+1} \). Use the quotient rule to find the exact values of the \( x \) coordinates of the stationary points of the curve. [7 marks]

4. a) Describe the graph transformation which maps \( y = f(x - 1) \) onto \( y = f(x + 3) \). [2 marks]
   b) Describe the graph transformation which maps \( y = f(2x) \) onto \( y = f(4x - 8) \). [4 marks]
   c) Find the coordinates of the image of the point \( P(3,-2) \) under the transformation described in b). [2 marks]

5. The functions \( f \) and \( g \) are defined with their respective domains by
   \[ f(x) = x^2 + 5x - 6, \quad \text{for } x > 1 \]
   \[ g(x) = |x + 10|, \quad \text{for all real values of } x \]
   a) Find the range of \( f \). [2 marks]
   b) The inverse of \( f \) is \( f^{-1} \).
      Find \( f^{-1}(x) \). Give your answer in its simplest form. [4 marks]
   c) i. Find \( fg(x) \). [1 mark]
      ii. Solve the equation \( fg(x) = 8 \) [6 marks]
6.

a) By using integration by parts twice, find
\[ \int x^2 \cos \frac{x}{2} \, dx \] [6 marks]

b) The curve below has equation \( y = x \sqrt{\cos \frac{x}{2}} \) for \( 0 \leq x \leq \pi \).

![Graph of the curve](image)

The region bounded by the curve and the \( x \)-axis is rotated through \( 2\pi \) radians about the \( x \)-axis to generate a solid. Find the exact value of the volume of the solid generated. [3 marks]

7. Use the substitution \( u = 4 - x^4 \) to find the exact value of
\[ \int_0^1 \frac{x^7}{4 - x^4} \, dx \] [6 marks]

8.

a) Show that the expression \( \frac{1 - \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} \) can be written as \( 2 \csc x \). [4 marks]

b) Hence solve the equation
\[ \frac{1 - \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} = 3 \cot^2 x - 5 \]
giving the values of \( x \) to the nearest degree in the interval \( 0^\circ \leq x < 360^\circ \). [6 marks]

c) Hence solve the equation
\[ \frac{1 - \cos \left( \frac{\theta}{2} - 20^\circ \right)}{\sin \left( \frac{\theta}{2} - 20^\circ \right)} + \frac{\sin \left( \frac{\theta}{2} - 20^\circ \right)}{1 - \cos \left( \frac{\theta}{2} - 20^\circ \right)} = 3 \cot^2 \left( \frac{\theta}{2} - 20^\circ \right) - 5 \]
giving the values of \( \theta \) to the nearest degree in the interval \( 0^\circ \leq \theta < 360^\circ \). [3 marks]

[Total: 75 marks]
1. Use Simpson's rule, with seven ordinates (six strips), to calculate an estimate for
\[
\int_0^{\pi/2} x^2 \cos x \, dx
\]
Give your answer to four significant figures.

\[ h = \frac{b - a}{n} = \frac{\pi/2 - 0}{6} = \frac{\pi}{12} \]

\[ x_0 = 0 \quad y_0 = 0^2 \cos 0 = 0 \]
\[ x_1 = \frac{\pi}{12} \quad y_1 = \left(\frac{\pi}{12}\right)^2 \cos \frac{\pi}{12} = 0.06620 \ldots \]
\[ x_2 = \frac{\pi}{6} \quad y_2 = \left(\frac{\pi}{6}\right)^2 \cos \frac{\pi}{6} = 0.23742 \ldots \]
\[ x_3 = \frac{\pi}{4} \quad y_3 = \left(\frac{\pi}{4}\right)^2 \cos \frac{\pi}{4} = 0.43617 \ldots \]
\[ x_4 = \frac{\pi}{3} \quad y_4 = \left(\frac{\pi}{3}\right)^2 \cos \frac{\pi}{3} = 0.54831 \ldots \]
\[ x_5 = \frac{5\pi}{12} \quad y_5 = \left(\frac{5\pi}{12}\right)^2 \cos \frac{5\pi}{12} = 0.44347 \ldots \]
\[ x_6 = \frac{\pi}{2} \quad y_6 = \left(\frac{\pi}{2}\right)^2 \cos \frac{\pi}{2} = 0 \]

\[
\int_0^{\pi/2} x^2 \cos x \, dx \approx \frac{1}{3} h \{ (y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) \}
\]

\[
= \frac{1}{3} \left(\frac{\pi}{12}\right) \{ (0 + 0) + 4(0.06620 \ldots + 0.43617 \ldots + 0.44347 \ldots) + 2(0.23742 \ldots + 0.54831 \ldots) \}
\]

\[ = 0.467305112 \ldots = \boxed{0.4673 \text{ to } 4 \text{ s.f.}} \]
2. A curve has equation \( y = \frac{\ln(e - 2x)}{2} \)

a) Find \( \frac{dy}{dx} \). \[ \text{[2 marks]} \]

b) Find an equation of the normal to the curve \( y = \frac{\ln(e - 2x)}{2} \) at the point on the curve where \( x = -e \). \[ \text{[4 marks]} \]

c) The curve \( y = \frac{\ln(e - 2x)}{2} \) crosses the line \( y = x \) at \( x = \alpha \).

i. Show that \( \alpha \) lies between 0 and 1. \[ \text{[2 marks]} \]

ii. Use the recurrence relation \( x_{n+1} = \frac{\ln(e - 2x_n)}{2} \) with \( x_1 = 0 \) to find the values of \( x_2 \) and \( x_3 \) to 4 significant figures. \[ \text{[2 marks]} \]

2.

a) Using chain rule:

\[
y = \frac{\ln(e - 2x)}{2} \implies \frac{dy}{dx} = \frac{1}{2} \times \frac{1}{e - 2x} \times -2 = \frac{-1}{e - 2x} = \frac{1}{2x - e}
\]

b) 

\[
\left( \frac{dy}{dx} \right)_{x=-e} = \frac{1}{2(-e) - e} = \frac{1}{-3e} \quad \implies \quad \text{Gradient of normal} = 3e
\]

\[
y_{x=-e} = \frac{\ln(e - 2(-e))}{2} = \frac{\ln 3e}{2}
\]

\[
y - y_1 = m(x - x_1) \implies \text{normal:} \quad y - \frac{\ln 3e}{2} = 3e(x + e)
\]

Or:

\[
y = 3ex + 3e^2 + \frac{1}{2}(1 + \ln 3)
\]

c) 

i. 

\( y = x \) crosses \( y = \frac{\ln(e - 2x)}{2} \) when \( x = \frac{\ln(e - 2x)}{2} \) \( \Leftrightarrow \frac{\ln(e - 2x)}{2} - x = 0 \)

\[
\frac{\ln(e - 0)}{2} - 0 = \frac{1}{2} > 0 \quad \text{and} \quad \frac{\ln(e - 2)}{2} - 1 \approx -1.1654 \ldots < 0
\]

Since there is a change of sign from \( x = 0 \) to \( x = 1 \), the root \( \alpha \) satisfies \( 0 < \alpha < 1 \).

ii. 

\( x_1 = 0 \implies x_2 = \frac{\ln(e - 0)}{2} = \frac{1}{2} = 0.5000 \text{ to } 4 \text{ s.f.} \implies x_3 = \frac{\ln(e - 2(0.5))}{2} = 0.2707 \text{ to } 4 \text{ s.f.} \)
3.

a) i. Differentiate \((3 - x^3)^\frac{3}{2}\) with respect to \(x\). [2 marks]

\[\frac{d}{dx} (3 - x^3)^\frac{3}{2} = \frac{3}{2} (3 - x^3)^\frac{1}{2}(-3x^2) = -\frac{9x^2(3 - x^3)^\frac{1}{2}}{2}\]

ii. Given that \(y = e^\frac{x}{2}(3 - x^3)^\frac{3}{2}\), find the exact value of \(\frac{dy}{dx}\) when \(x = 0\). [3 marks]

\[\frac{dy}{dx} = e^\frac{x}{2} \left( -\frac{9x^2(3 - x^3)^\frac{1}{2}}{2} \right) + (3 - x^3)^\frac{3}{2} \left( \frac{1}{2} e^\frac{x}{2} \right)\]

\[\left( \frac{dy}{dx} \right)_{x=0} = e^0 \left( -\frac{9(0)(3 - 0)^\frac{1}{2}}{2} \right) + (3 - 0)^\frac{3}{2} \left( \frac{1}{2} e^0 \right) = \frac{3\sqrt{3}}{2}\]

b) A curve has equation \(y = \frac{x-1}{x^2+1}\). Use the quotient rule to find the exact values of the \(x\) coordinates of the stationary points of the curve. [7 marks]

\[y = \frac{x-1}{x^2+1} \Rightarrow \frac{dy}{dx} = \frac{(x^2+1)(1) - (x-1)(2x)}{(x^2+1)^2}\]

\[\frac{dy}{dx} = 0 \Rightarrow (x^2+1)(1) - (x-1)(2x) = 0 \Rightarrow -x^2 + 2x + 1 = 0\]

\[\Rightarrow x^2 - 2x - 1 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}\]
4.
a) Describe the graph transformation which maps $y = f(x - 1)$ onto $y = f(x + 3)$. [2 marks]

b) Describe the graph transformation which maps $y = f(2x)$ onto $y = f(4x - 8)$. [4 marks]

c) Find the coordinates of the image of the point $P(3, -2)$ under the transformation described in b). [2 marks]

4.
a) Translation by vector $\begin{bmatrix} -4 \\ 0 \end{bmatrix}$.

b) Stretch in the x direction by scale factor $\frac{1}{2}$ followed by a translation by vector $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$.
OR:
Translation by vector $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ followed by a stretch in the x direction by scale factor $\frac{1}{2}$.

c) $(3, -2) \rightarrow \left(\frac{3}{2}, -2\right) \rightarrow \left(\frac{7}{2}, -2\right)$

OR:
$(3, -2) \rightarrow (7, -2) \rightarrow \left(\frac{7}{2}, -2\right)$
5. The functions $f$ and $g$ are defined with their respective domains by

\[ f(x) = x^2 + 5x - 6, \quad \text{for } x > 1 \]
\[ g(x) = |x + 10|, \quad \text{for all real values of } x \]

a) Find the range of $f$. \[ \text{[2 marks]} \]

b) The inverse of $f$ is $f^{-1}$. \[ \text{[4 marks]} \]

Find $f^{-1}(x)$. Give your answer in its simplest form.

c) i. Find $f g(x)$.

\[ \text{[1 mark]} \]

ii. Solve the equation $f g(x) = 8$ \[ \text{[6 marks]} \]

5. a)

\[ f(x) = x^2 + 5x - 6 = (x + 6)(x - 1) \]

Therefore the graph $y = f(x)$ crosses the $x$-axis at the points $x = -6$ and $x = 1$ (and the $y$-axis at $y = -6$). From the graph it is apparent that the range is:

\[ f(x) > 0 \]

b)

\[ f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - 6 = \left(x + \frac{5}{2}\right)^2 - \frac{49}{4} \]

Let $y = f(x)$:

\[ \left(x + \frac{5}{2}\right)^2 - \frac{49}{4} = y \quad \Rightarrow \quad \left(x + \frac{5}{2}\right)^2 = y + \frac{49}{4} \quad \Rightarrow \quad x + \frac{5}{2} = \pm \sqrt{y + \frac{49}{4}} \quad \Rightarrow \quad x = -\frac{5}{2} \pm \sqrt{y + \frac{49}{4}} \]

Reflecting in $y = x$:

\[ y = -\frac{5}{2} \pm \sqrt{x + \frac{49}{4}} \]

However, since the range of $f^{-1}$ is the domain of $f$, it is $f^{-1}(x) > 1$. Therefore we reject the $-ve$ root:

\[ f^{-1}(x) = -\frac{5}{2} + \sqrt{x + \frac{49}{4}} \quad \text{or} \quad f^{-1}(x) = -\frac{5}{2} + \sqrt{\frac{4x + 49}{2}} \]

c) i.

\[ f g(x) = f(g(x)) = f(|x + 10|) = (|x + 10|)^2 + 5|x + 10| - 6 = (x + 10)^2 + 5|x + 10| - 6 \]

ii.

\[ (x + 10)^2 + 5(x + 10) - 6 = 8 \quad \text{or} \quad (x + 10)^2 - 5(x + 10) - 6 = 8 \]

\[ (x + 10)^2 + 5(x + 10) - 14 = 0 \quad \Rightarrow \quad x^2 + 20x + 100 + 5x + 50 - 14 = 0 \quad \Rightarrow \quad x^2 + 25x + 136 = 0 \]

\[ \Rightarrow \quad (x + 8)(x + 17) = 0 \quad \Rightarrow \quad x = -8 \quad \text{or} \quad x = -17 \]

\[ (x + 10)^2 - 5(x + 10) - 14 = 0 \quad \Rightarrow \quad x^2 + 20x + 100 - 5x - 50 - 15 = 0 \quad \Rightarrow \quad x^2 + 15x + 36 = 0 \]

\[ \Rightarrow \quad (x + 3)(x + 12) = 0 \quad \Rightarrow \quad x = -3 \quad \text{or} \quad x = -12. \quad \text{All solutions: } x = -17, -12, -8 \text{ or } -3 \]
6.

a) By using integration by parts twice, find

\[ \int x^2 \cos \frac{x}{2} \, dx \]  

[6 marks]

b) The curve below has equation \( y = x \sqrt{\cos \frac{x}{2}} \) for \( 0 \leq x \leq \pi \).

The region bounded by the curve and the \( x \)-axis is rotated through \( 2\pi \) radians about the \( x \)-axis to generate a solid. Find the exact value of the volume of the solid generated.

[3 marks]

6.

a) Integrating \( \int x^2 \cos \frac{x}{2} \, dx \) by parts:

\[ u = x^2 \quad \frac{dv}{dx} = \cos \frac{x}{2} \]

\[ du = 2x \quad v = 2 \sin \frac{x}{2} \]

\[ \int x^2 \cos \frac{x}{2} \, dx = 2x^2 \sin \frac{x}{2} - \int 4x \sin \frac{x}{2} \, dx = 2x^2 \sin \frac{x}{2} - 4 \int x \sin \frac{x}{2} \, dx \]

Integrating \( \int x \sin \frac{x}{2} \, dx \) by parts:

\[ u = x \quad \frac{dv}{dx} = \sin \frac{x}{2} \]

\[ du = 1 \quad v = -2 \cos \frac{x}{2} \]

\[ \int x \sin \frac{x}{2} \, dx = -2x \cos \frac{x}{2} - \int -2 \cos \frac{x}{2} \, dx = -2x \cos \frac{x}{2} + 4 \sin \frac{x}{2} + K \]

Substituting into expression for \( \int x^2 \cos \frac{x}{2} \, dx \) and incorporating the constant of integration:

\[ \int x^2 \cos \frac{x}{2} \, dx = 2x^2 \sin \frac{x}{2} - 4 \left( -2x \cos \frac{x}{2} + 4 \sin \frac{x}{2} \right) + C = 2x^2 \sin \frac{x}{2} + 8x \cos \frac{x}{2} - 16 \sin \frac{x}{2} + C \]

b) \( V = \pi \int_0^\pi y^2 \, dx = \pi \int_0^\pi x^2 \cos \frac{x}{2} \, dx = \pi \left[ 2x^2 \sin \frac{x}{2} + 8x \cos \frac{x}{2} - 16 \sin \frac{x}{2} \right]_0^\pi = \pi [(2\pi^2 - 16) - (0)] \]

\[ \Rightarrow \quad V = 2\pi(\pi^2 - 8) \]
7. Use the substitution \( u = 4 - x^4 \) to find the exact value of
\[
\int_0^1 \frac{x^7}{4 - x^4} \, dx
\]

[6 marks]

\[
7. \quad u = 4 - x^4 \quad \Rightarrow \quad \frac{du}{dx} = -4x^3 \quad \Rightarrow \quad -\frac{1}{4} \, du = x^3 \, dx
\]

\[
x = 0 \quad \Rightarrow \quad u = 4 - 0 = 4 \quad \text{and} \quad x = 1 \quad \Rightarrow \quad u = 4 - 1^4 = 3
\]

\[
\int_0^1 \frac{x^7}{4 - x^4} \, dx = -\frac{1}{4} \int_4^3 \frac{x^4}{u} \, du = -\frac{1}{4} \int_4^3 \frac{4 - u}{u} \, du = -\frac{1}{4} \int_4^3 \frac{4}{u} - 1 \, du = -\frac{1}{4} \left[ 4 \ln u - u \right]_4^3
\]

\[
= -\frac{1}{4} \left[ (4 \ln 3 - 3) - (4 \ln 4 - 4) \right] = -\frac{1}{4} \left[ 4(\ln 3 - \ln 4) + 1 \right] = \ln 4 - \ln 3 - \frac{1}{4}
\]
8.

a) Show that the expression \( \frac{1-\cos x}{\sin x} + \frac{\sin x}{1-\cos x} \) can be written as \( 2 \csc x \). [4 marks]

b) Hence solve the equation

\[
\frac{1-\cos x}{\sin x} + \frac{\sin x}{1-\cos x} = 3 \cot^2 x - 5
\]

giving the values of \( x \) to the nearest degree in the interval \( 0^\circ \leq x < 360^\circ \). [6 marks]

c) Hence solve the equation

\[
\frac{1-\cos (\theta - 20^\circ)}{\sin (\theta - 20^\circ)} + \frac{\sin (\theta - 20^\circ)}{1-\cos (\theta - 20^\circ)} = 3 \cot^2 (\theta - 20^\circ) - 5
\]

giving the values of \( \theta \) to the nearest degree in the interval \( 0^\circ \leq \theta < 360^\circ \). [3 marks]

[Total: 75 marks]