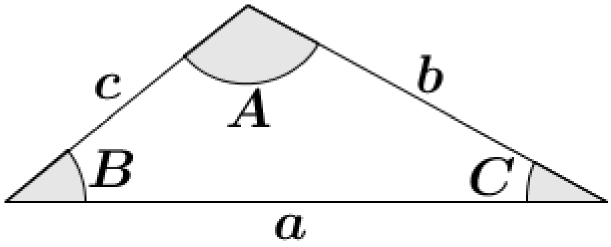
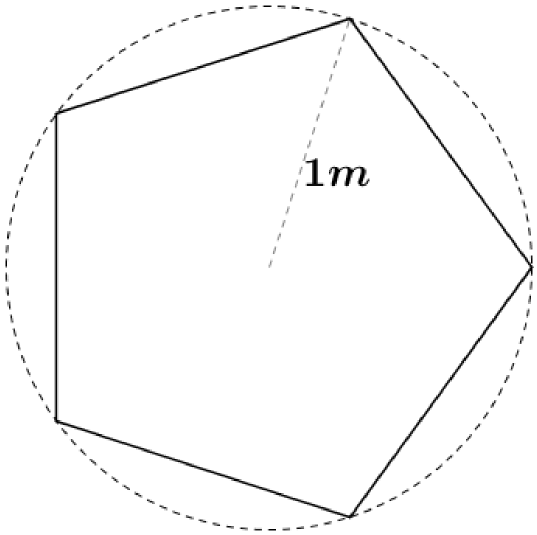
Area of a Regular Polygon Investigation

For a triangle labelled as shown, the area can be found using the formula:

$$Area = \frac{1}{2}ab\sin C$$



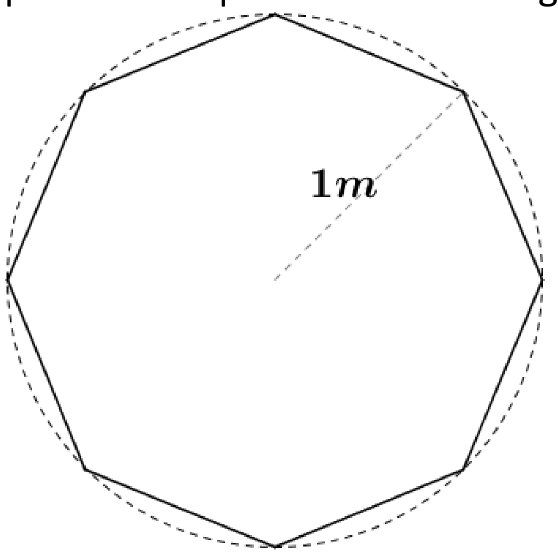
A regular pentagon is inscribed in a circle of radius $1m$ as shown:



1. By splitting it into triangles, find the area of the whole pentagon.

Hint: the angles at the centre must add up to $360°$, and every triangle made by joining corners to the middle is isosceles.

The process is repeated for an octagon.



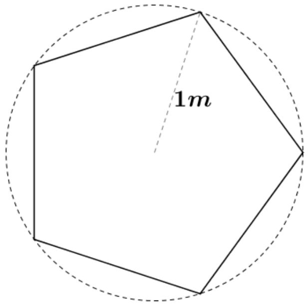
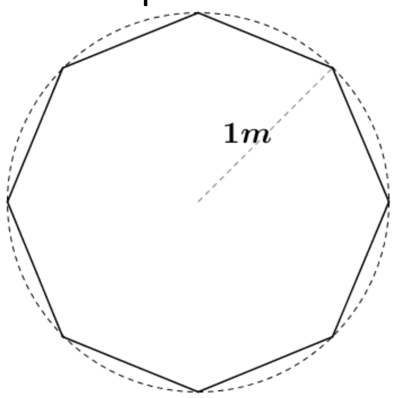
2. Use the same process to calculate the area of the octagon.

3. Create a formula that calculates the area for a polygon with n sides.

4. Use your formula to find the area of a 100-sided shape:

5. What do you notice about your answer? Why do you think this is the case?

Area of a Regular Polygon Investigation SOLUTIONS

A regular pentagon is inscribed in a circle of radius $1m$ as shown: 	1. By splitting it into triangles, find the area of the whole pentagon. *Area of one triangle*: $\frac{1}{2} \times 1 \times 1 \times \sin 72 \approx 0.4755$ *Area of 5 triangles*: $5 \times 0.4755 \ldots \approx \mathbf{2.3776cm^2}$
The process is repeated for an octagon. 	2. Use the same process to calculate the area of the octagon. *Area of one triangle*: $\frac{1}{2} \times 1 \times 1 \times \sin 45 \approx 0.3536$ *Area of 8 triangles*: $8 \times 0.3535 \ldots \approx \mathbf{2.8284m^2}$

3. Create a formula that calculates the area for a polygon with n sides.

Angle at centre: $\frac{360}{n}$. Area of each triangle: $\frac{1}{2} \times 1 \times 1 \times \sin\frac{360}{n} = \frac{1}{2}\sin\frac{360}{n}$.

Area of the entire polygon: $n \times \frac{1}{2}\sin\frac{360}{n} = \frac{n}{2}\sin\frac{360}{n}$. In general: $\frac{r^2 n}{2}\sin\frac{360}{n}$.

4. Use your formula to find the area of a 100-sided shape:

$$\frac{100}{2}\sin\frac{360}{100} = 50\sin 3.6 \approx 3.139m^2$$

5. What do you notice about your answer? Why do you think this is the case?

The value is very close to π. This is because the more sides a polygon has, the closer its area gets to that of the circle it is inscribed within.

To get an even closer approximation, try $\frac{1000}{2}\sin\frac{360}{1000}$ or $\frac{10000}{2}\sin\frac{360}{10000}$...