Area of a Regular Polygon Investigation

For a triangle labelled as shown, the area can be found using the formula:

\[ \text{Area} = \frac{1}{2}ab \sin C \]

A regular pentagon is inscribed in a circle of radius 1m as shown:

1. By splitting it into triangles, find the area of the whole pentagon.

\[ \text{Hint: the angles at the centre must add up to 360°, and every triangle made by joining corners to the middle is isosceles.} \]

The process is repeated for an octagon.

2. Use the same process to calculate the area of the octagon.

3. Create a formula that calculates the area for a polygon with \( n \) sides.

4. Use your formula to find the area of a 100-sided shape:

5. What do you notice about your answer? Why do you think this is the case?
A regular pentagon is inscribed in a circle of radius 1m as shown:

1. By splitting it into triangles, find the area of the whole pentagon.

\[
\text{Area of one triangle:} \quad \frac{1}{2} \times 1 \times 1 \times \sin 72 \approx 0.4755 \\
\text{Area of 5 triangles:} \quad 5 \times 0.4755 \ldots \approx 2.3776 \text{cm}^2
\]

The process is repeated for an octagon.

2. Use the same process to calculate the area of the octagon.

\[
\text{Area of one triangle:} \quad \frac{1}{2} \times 1 \times 1 \times \sin 45 \approx 0.3536 \\
\text{Area of 8 triangles:} \quad 8 \times 0.3535 \ldots \approx 2.8284 \text{m}^2
\]

3. Create a formula that calculates the area for a polygon with \( n \) sides.

Angle at centre: \( \frac{360}{n} \). Area of each triangle: \( \frac{1}{2} \times 1 \times 1 \times \sin \left( \frac{360}{n} \right) = \frac{1}{2} \sin \left( \frac{360}{n} \right) \).

Area of the entire polygon: \( n \times \frac{1}{2} \sin \left( \frac{360}{n} \right) = \frac{n}{2} \sin \left( \frac{360}{n} \right) \). In general: \( \frac{r^2 \pi}{2} \sin \left( \frac{360}{n} \right) \).

4. Use your formula to find the area of a 100-sided shape:

\[
\frac{100}{2} \sin \frac{360}{100} = 50 \sin 3.6 \approx 3.139 \text{m}^2
\]

5. What do you notice about your answer? Why do you think this is the case? The value is very close to \( \pi \). This is because the more sides a polygon has, the closer its area gets to that of the circle it is inscribed within.

To get an even closer approximation, try \( \frac{1000}{2} \sin \frac{360}{1000} \) or \( \frac{10000}{2} \sin \frac{360}{10000} \)...