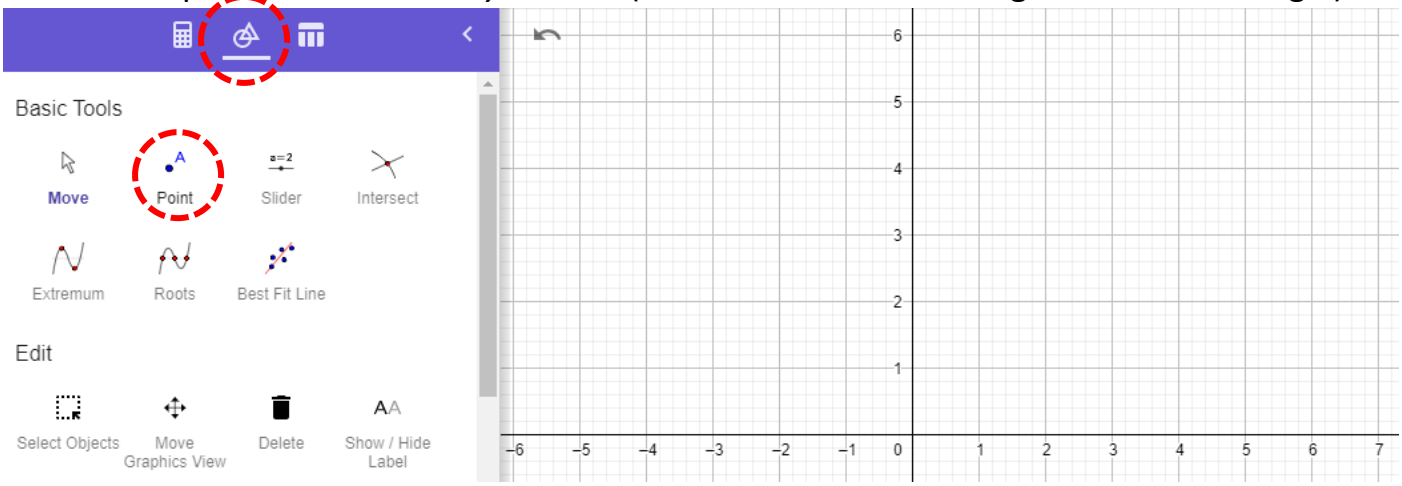
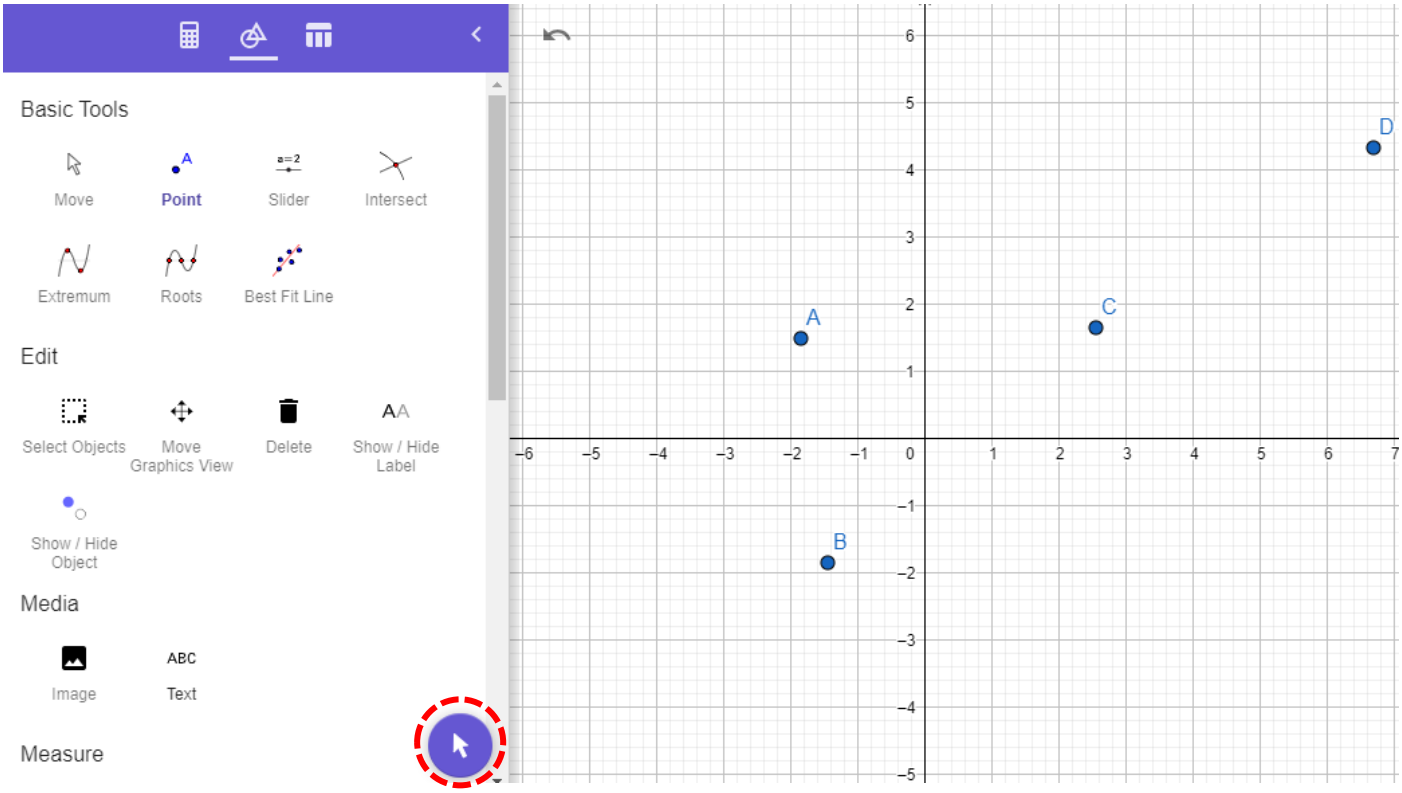
Calculus: Investigating Functions with GeoGebra

GeoGebra has a bunch of cool tools you can use to analyse functions.

To begin with, let's create a polynomial that is easy to manipulate: Choose **Point** from the **Basic Tools** part of the *Geometry* section (the icon is an intersecting circle and a triangle):

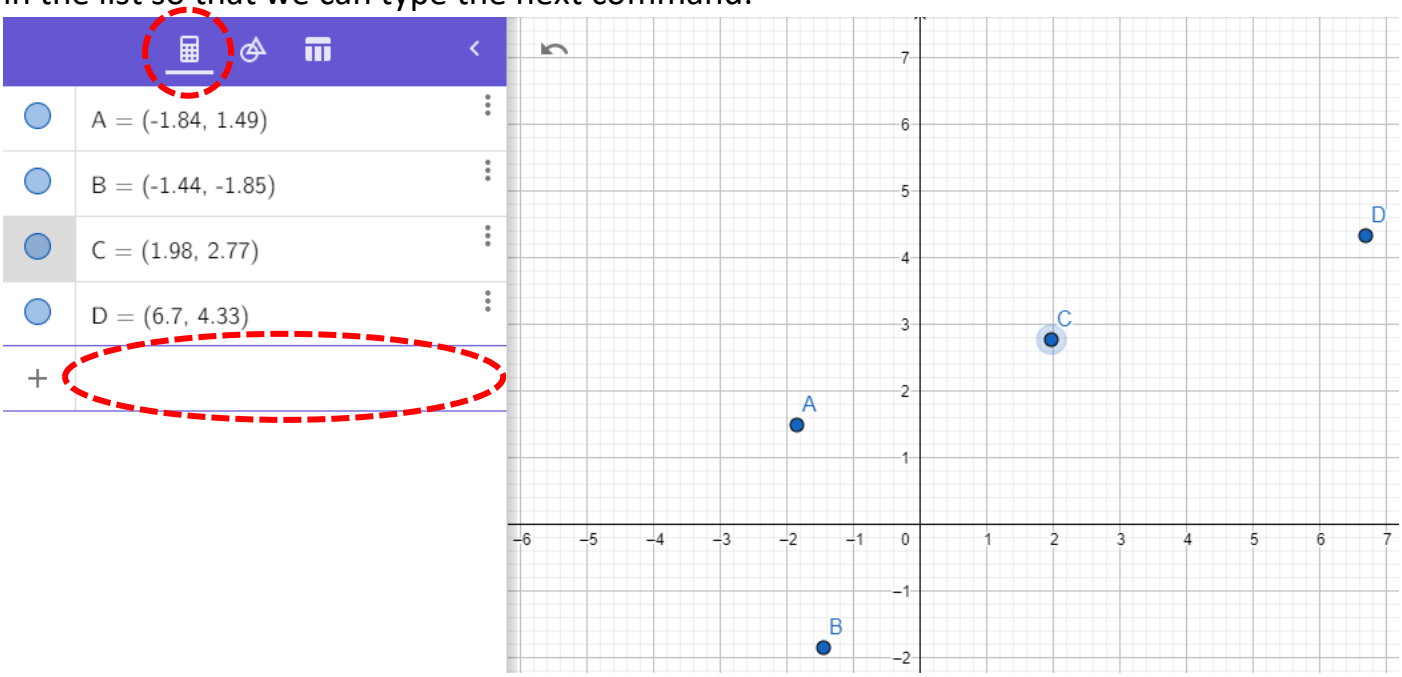


Click four random places on the grid to the right (for now, avoid the axes, or GeoGebra will assume you want the points anchored to them, and it restricts your flexibility):



When you're done, clicking on the blue circle with the mouse pointer icon will allow you to click and drag to reposition the points, or click and drag anywhere else to move around.

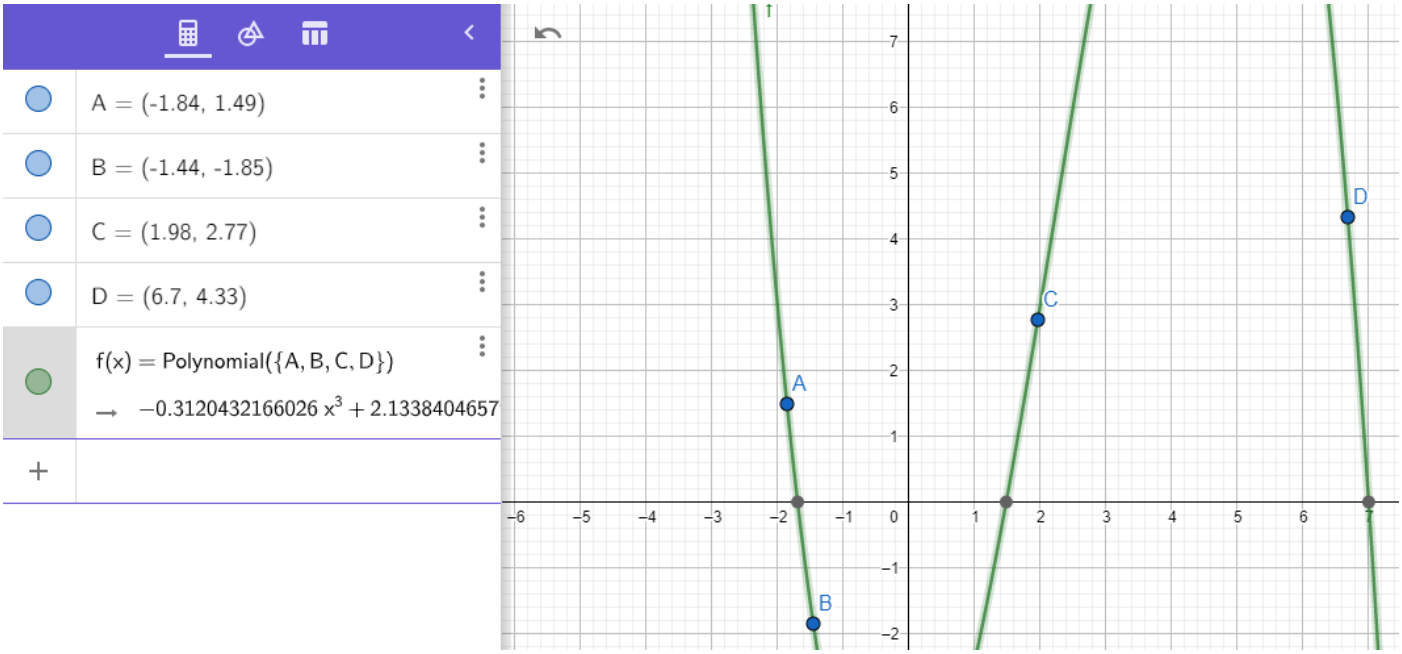
Next, select the *Algebra* section (calculator icon at the top), and click in the next free space in the list so that we can type the next command:



Type the following, then press Enter:

polynomial(A,B,C,D)

This produces the simplest possible polynomial graph passing through the given points (two points uniquely defines a line, three a quadratic, four a cubic, etc). The equation is shown on the left, and the graph drawn on the right. Try moving your points to change the cubic.



Turn over to see how to analyse the **derivative** and the **integral** of your function.

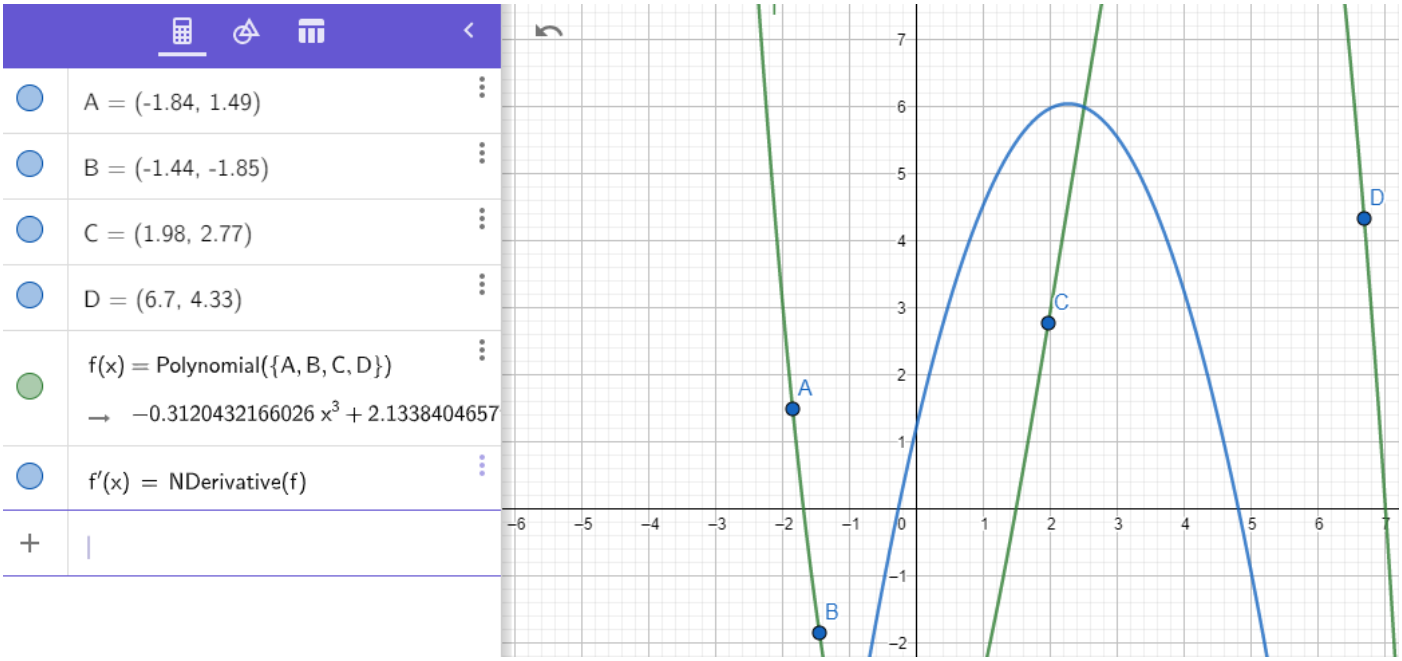
The **derivative**:

The derivative of a function shows the rate of change of that function (its gradient) for any given x value. GeoGebra can sketch the derivative function directly for you: just type the following into the next available input box, and hit Enter:

f'(x)

This generates the gradient function for the function f, and graphs it for you.

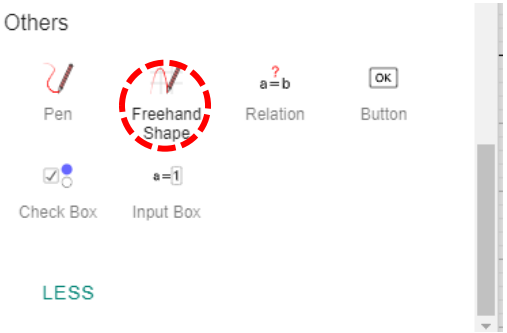
Tip: one apostrophe gives the first derivative, two the second derivative, and so on.



By moving the original points, you will be able to see changes to the original function and to its derivative. *Are there any arrangements of your four points that don't produce a cubic?*

If you click on the coloured circle beside a function in the pane on the left, you can show or hide it. *Try hiding the original function $f(x)$, moving your four points around, and then predicting what the cubic will look like based on the graph of its first derivative.*

Another neat way to make a custom function is the **Freehand Shape** tool: scroll to the bottom of the *Geometry* menu, click **MORE**, then select **Freehand Shape** and draw a curve.

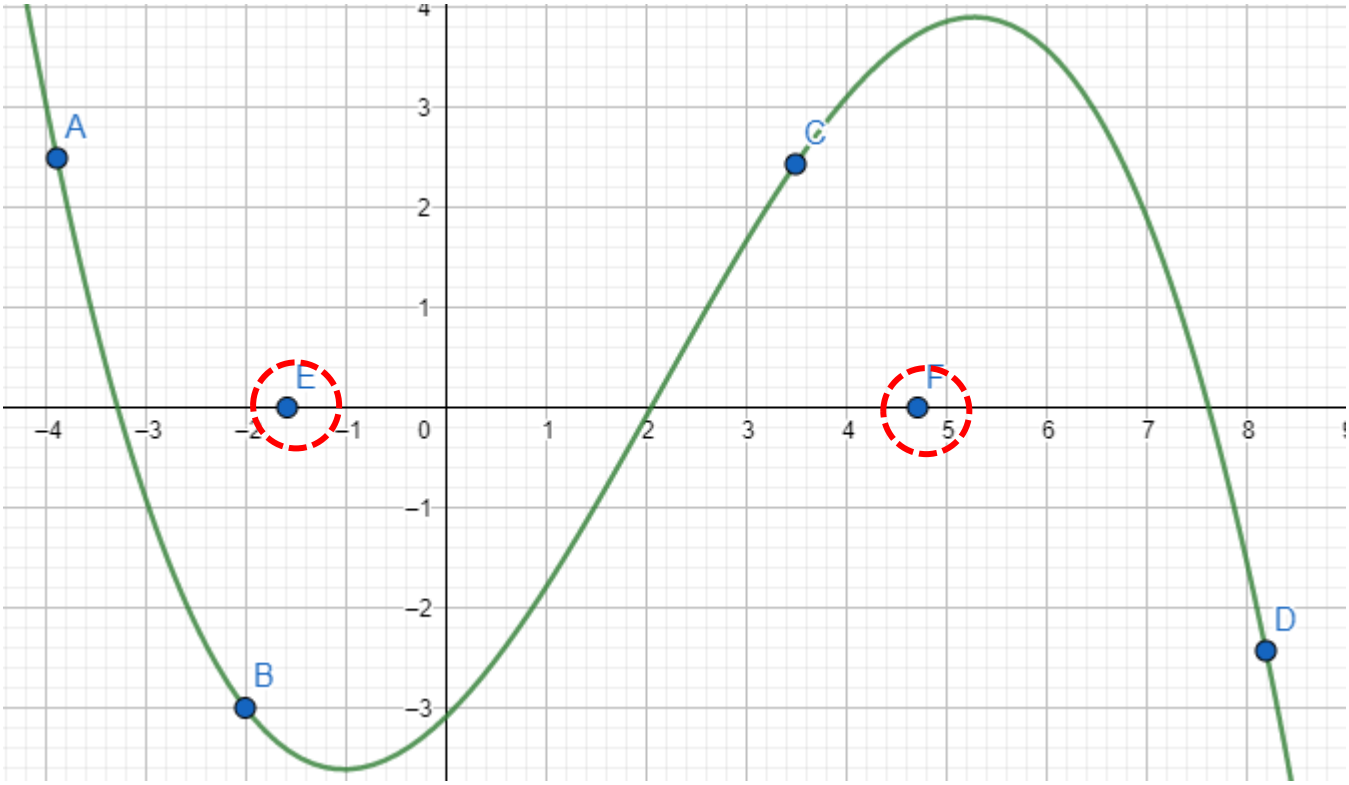


GeoGebra will give this a label like $g(x)$ in the *Algebra* menu, although since it is defined by where you choose to draw rather than through elementary mathematical functions, GeoGebra won't be able to give you its derivative directly. If you plot enough points on your curve, however, you can fit a polynomial function to it in the same way that you made a cubic function above.

The **integral**:

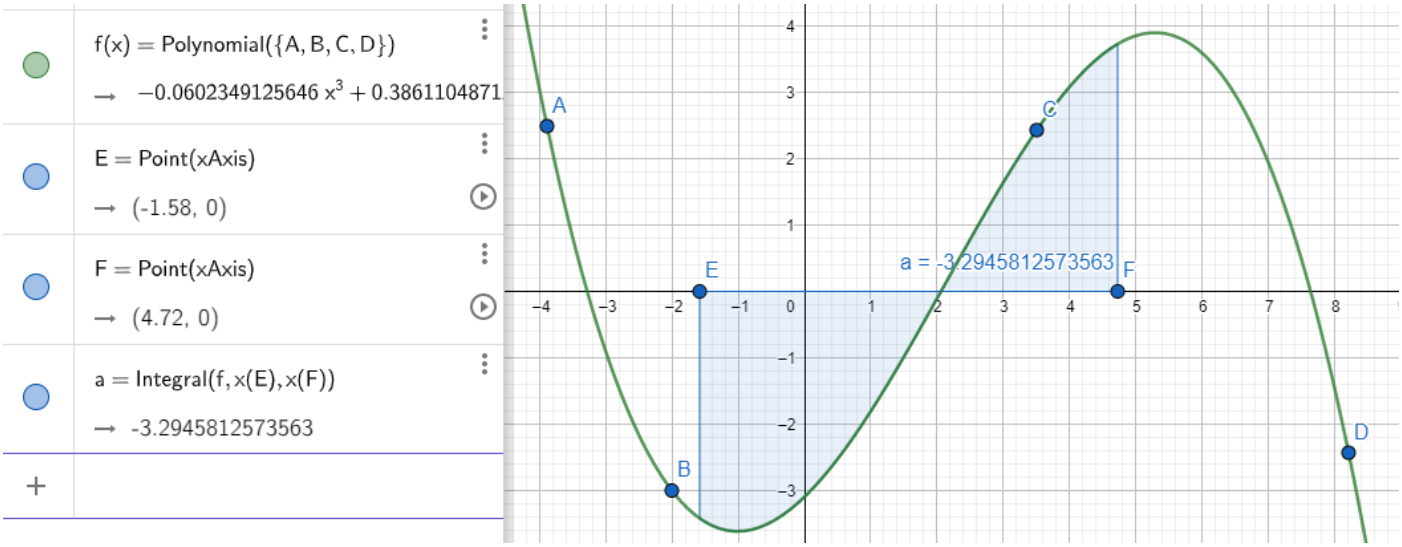
The integral of a function gives the area enclosed by the function over a given interval of x values. GeoGebra can both calculate this and represent it for any given curve.

First, we need to define the limits between which we want to integrate, so create two new points, but this time locate them on the x-axis, so GeoGebra anchors them to that axis:



The **integral** function in GeoGebra takes three arguments: the function itself, and two x values to integrate between. Since we want to use the x coordinates of the points E and F, we use the **x** function to extract the x coordinate:

integral(f, x(E), x(F))



Move E and F around and see how the value of the integral changes. *Can you make it equal to zero? Less than zero? Are there any local maximum or minimum values?*